University of Texas at Austin
Macro Comp, June 2017
Answer each question, they are worth 45 points a piece.

1. General Equilibrium and Optimality in OLG

Consider the two-age OLG model with utility of the form $U^t(c_t^t, c_{t+1}^t) = c_{t+1}^t$ and endowment process $e_t^t = G^t, e_{t+1}^t = 0$, where $G > 1$. That is, households only care about consumption when they are old and only receive an endowment of consumption goods when young, which is growing over time. As usual, time starts at $t = 0$, at which point there is a group of initial old who have endowment $e_0^{-1} = 0$ of consumption goods. Throughout this question, assume that there is *no free disposal*, so that markets must clear exactly.

1. Define a sequential markets equilibrium where households buy a_t units of an Arrow Security when young at period t and receive $a_t(1+r_{t+1})$ from those purchases when old in period $t+1$.

2. Fully solve for the above equilibrium (ie, tell me what the equilibrium interest rate has to be in each period, along with the consumption and asset allocations).

3. List the Pareto Efficient allocation(s) for this economy.

4. Now suppose that each initial old household is endowed with a single handsome cat figurine in $t = 0$.[1] The figurines contribute nothing to utility for anybody, but lasts forever. Define an equilibrium in which the figurines can be bought and sold and show that there is an equilibrium with a positive price. What is the growth rate of the price of handsome cat figurines?

5. Suppose that some people are worried about a handsome cat figurine *bubble* and a policy is implemented to make it illegal for the price of cat figurines to grow over time. What happens to consumption and welfare in the cat figurine price-ceiling economy relative to the laissez-faire economy as $t \to \infty$?

[1] A figurine is a small statue, like a toy.

2. Tax Distorted Competitive Equilibrium

Consider the Neo-Classical growth model in which a representative household has period utility function over consumption and labor supply $u(c,h) = \log(c) - \phi\frac{\epsilon}{1+\epsilon}h^{\frac{1+\epsilon}{\epsilon}}$ with $\epsilon > 0$ and constant discount factor β. Assume that there is a representative firm with production function $Y_t = AK_t^{\alpha}H_t^{1-\alpha}$. Capital depreciates at rate $\delta = 100\%$.

A. Solve for a competitive equilibrium in this economy.

Now assume that the government taxes all sources of income at constant rate τ_t each period and uses the proceeds to buy goods $G_t = \tau_t\Big(w_tH_t + r_tK_t\Big)$ so that the budget balances in every period.

B. Define the TDCE for this economy and show how GDP depends on $\big(\tau_t\big)_{t=0}^{\infty}$ at each point in time.

C. Suppose that $\tau_t \to \tau$. Show that there is a steady-state equilibrium and solve for the marginal product of labor, $w(\tau)$, as a function of τ.

D. Draw the "static" labor-Laffer Curve for the steady state economy. That is, denote revenue as $\tilde{R}(\tau) = \tau\tilde{w}h(\tau)$, where $\tilde{w}$ corresponds to the steady-state wage when $\tau = 0$. What tax rate maximizes $\tilde{R}$ (i.e. where is the top of the Laffer Curve)?

E. Now create the "dynamic" Laffer Curve by using the endogenous steady-state wage to plot $R(\tau) = \tau w(\tau)h(\tau)$. Is the revenue maximizing rate above, below, or the same as the "static" curve from part D?

3. Two-sector business cycle model (45 points)

Consider a closed-economy two-sector business cycle model with perfect factor mobility, where one sector produces a consumption good and the other produces an investment good. There is no growth for simplicity.

Preferences

The representative household's expected discounted utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right]$$

where C_t is consumption and N_t is hours. Moreover, $0 < \beta < 1$ is the discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, and $\phi > 0$ is the inverse of the Frisch elasticity of labor supply.

Production technology in the consumption good sector

The consumption good sector's production function is given by

$$Y_{C,t} = A_{C,t} K_t^{\alpha} N_{C,t}^{1-\alpha}$$

where $Y_{C,t}$ is consumption good output, $N_{C,t}$ is labor, K_t is capital, and $A_{C,t}$ is a random, sector-specific stationary productivity shock. Moreover, $0 < \alpha < 1$ determines the weight of the capital input.

Production technology in the investment good sector

The investment good sector's production function is given by

$$Y_{I,t} = A_{I,t} N_{I,t}$$

where $Y_{I,t}$ is investment good output, $N_{I,t}$ is labor, and $A_{I,t}$ is a random, sector-specific stationary productivity shock.

Capital accumulation technology

The evolution of capital is given by

$$K_{t+1} = I_t + (1-\delta) K_t$$

where I_t is investment, $0 < \delta < 1$ is the rate of depreciation, and $K_0 > 0$ is given.

Resource constraints

The resource constraints for the two goods are given by

$$Y_{C,t} = C_t \text{ and } Y_{I,t} = I_t.$$

Labor mobility and market clearing

Labor is perfectly mobile between the two sectors and the market clearing condition is

$$N_t = N_{C,t} + N_{I,t}$$

(a) Formulate a price-taking version of the model where the representative household owns the capital stock.

(b) Show that the equilibrium allocations of the formulation in (a) are the same as the solution to the planner's problem for the model.

(c) Consider the price-taking version you formulated in (a). There, introduce the following change in the consumption good sector's production function

$$Y_{C,t} = X_{C,t} K_t^{\alpha} N_{C,t}^{1-\alpha}$$

where

$$X_{C,t} = A_{C,t} K_t^{\gamma_K - \alpha} N_{C,t}^{\gamma_L - (1-\alpha)}$$

is taken *as given* by the firm and $\gamma_K, \gamma_L > 0$ are parameters. Everything else about the environment remains the same as in (a). (If it is helpful, you can think here of a large number of identical firms, measure 1, who take $X_{C,t}$ as given when solving their maximization problem.)

Argue carefully whether the equilibrium allocations of the price-taking version and the planner's version for this modified model will coincide.

(You do not have to, necessarily, completely set-up and solve for the two equilibria. But you do have to defend your answer completely and rigorously using economic arguments.)

4. Optimal monetary and fiscal policy (45 points)

The government's objective is to minimize the loss function

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\pi_{t+j}^2 + \lambda_y y_{t+j}^2 + \lambda_T T_{t+j}^2\right]$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t,$$

$$y_t = E_t y_{t+1} - (i_t - E_t \pi_{t+1}) + \varepsilon_t,$$

and

$$b_t = \beta^{-1} b_{t-1} - \beta^{-1} \pi_t + i_t - \varphi T_t.$$

Here, E_t is the mathematical expectation operator conditional on period-t information, π_t is inflation, y_t is output, i_t is the one-period nominal interest rate, T_t is taxes, and b_t is the real value of one-period nominal government debt. Moreover, $\beta \in (0,1)$ and λ_y, λ_T, κ, $\varphi > 0$ are model parameters. Finally, ε_t is an $AR(1)$ shock that follows

$$\varepsilon_t = \rho \varepsilon_{t-1} + \upsilon_t$$

where $\rho \in [0,1)$ and υ_t is an *iid*, mean zero innovation.

The government's instruments are i_t (monetary) and T_t (fiscal), which are chosen after υ_t is realized at the beginning of the period.

(a) Suppose that the government *can* commit at date 0 to a fully contingent path for i_t and T_t.

(i) Characterize, as far as you can, the solution to the optimal monetary and fiscal policy problem with commitment.

(ii) Given your characterization in (i) above, what process is followed by T_t in equilibrium? What about b_t?

(b) Now suppose that the government *cannot* commit at date 0 to a contingent path for i_t and T_t and instead chooses i_t and T_t at each date. The solution concept for this no-commitment case is that of a Markov-perfect equilibrium.

(iii) Argue carefully whether the no-commitment equilibrium will differ from the commitment equilibrium. (You do not have to, necessarily, completely set-up and solve for equilibrium.)

(iv) Given your answer in (iii) above, will the process followed by T_t and b_t, in particular, differ between the commitment and no-commitment equilibrium? Defend your answer completely, using economic reasoning to justify whether and how exactly the equilibrium solutions are different or not.