ECON 387
Spring 2018–II

Comprehensive Exam

There are 4 questions and a total of 180 points.

1. Home production in a RBC model (45 points)

Consider the following closed-economy RBC model, where in addition to the standard market based sector, there is a home production (or non-market based) sector that the household owns/operates. There is no growth for simplicity.

Preferences

The representative household's expected discounted utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right)$$

where E_0 is the conditional expectation operator, C_t is consumption, L_t is leisure, and $0 < \beta < 1$ is the discount factor. The period utility $u(\cdot)$ is strictly increasing, concave, and twice continuously differentiable. Here, C_t is a bundle of the market good and home produced (non-market) goods, given by the aggregator

$$C_t = \left(C_t^H\right)^{\gamma} \left(C_t^M\right)^{1-\gamma}$$

where C_t^H is consumption of the home-produced (non-market) good, C_t^M is consumption of the market good, and $0 < \gamma < 1$ determines the consumption-share of the home-produced good.

Production Technologies

In this economy, output (Y_t^H) of the home-produced good is produced using a production function

$$Y_t^H = A_t^H F(K_t^H, N_t^H)$$

where K_t^H is (pre-determined) capital and N_t^H is labor used for home-production while A_t^H is a stationary productivity shock.

In this economy, output (Y_t^M) of the market good is produced using a production function

$$Y_t^M = A_t^M F(K_t^M, N_t^M)$$

where K_t^M is (pre-determined) capital and N_t^M is labor used for production of the market good while A_t^M is a stationary productivity shock.

The production function $F(\cdot)$ is twice continuously differentiable, concave, and homogenous of degree one. It also satisfies the standard limiting conditions (the Inada conditions).

Accumulation Technologies

The evolution of the two types of capital K_t^H and K_t^M are given by

$$K_{t+1}^H = I_t^H + (1-\delta)\, K_t^H$$
$$K_{t+1}^M = I_t^M + (1-\delta)\, K_t^M$$

where I_t^H is investment in home-production (non-market) and I_t^M is investment in the market sector while $0 < \delta < 1$ is the rate of depreciation.

Factor Mobility

The two factors of production, labor and capital, are perfectly mobile/substitutable between use in the production of the home-produced (non-market) or the market good.

Resource Constraints

The total amount of time that the household has can be split into work in home-production or production of the market good and leisure. Normalizing the total amount of time each period to be 1, the time constraint is

$$N_t^H + N_t^M + L_t = 1.$$

The resource constraint for the home-produced good is

$$Y_t^H = C_t^H$$

while the resource constraint for the market good is

$$Y_t^M = C_t^M + I_t^H + I_t^M$$

as it can be used for consumption as well as investment in both the production of the home and market good.

(i) Formulate a price-taking version of the above model in which *the representative household owns the two capital stocks and technology for home-production* while the *representative firm produces the market good while renting capital and hiring labor in competitive markets.*

(ii) Define the competitive equilibrium based on your formulation in (i) above and derive all the conditions that characterize it.

(iii) Consider a specific formulation for the period utility

$$u\,(C_t, L_t) \equiv \log C_t + \theta \log(1 - N_t^H - N_t^M).$$

What does this formulation imply about how the household values working in the market vs. home production sector?

2. Optimal monetary policy without commitment in a sticky price model (45 points)

The central bank's objective is to minimize the loss function

$$\frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^j\left[\phi_\pi\pi_{t+j}^2+\phi_x x_{t+j}^2+\phi_i i_{t+j}^2\right]$$

subject to

$$\pi_t=\beta E_t\pi_{t+1}+\kappa x_t+\varepsilon_{\pi,t},$$

$$(x_t-\lambda x_{t-1})=E_t\left(x_{t+1}-\lambda x_t\right)-\left(i_t-E_t\pi_{t+1}\right)+\varepsilon_{x,t}$$

where E_t is the conditional expectation operator, i_t is the central bank's instrument, π_t and x_t are other endogenous model variables, and $0<\beta<1$, $0<\lambda<1$, $\kappa>0$, $\phi_\pi>0, \phi_x>0, \phi_i>0$ are model parameters. The central bank takes actions after the shocks $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$ are realized. The shocks $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$ are iid over time and have unit variance.

(i) Suppose that the central bank cannot credibly commit and, thus, chooses i_t at each date. Characterize, as far as you can, the (Markov-perfect) solution to the optimal monetary policy problem above without commitment.

(iii) Next, consider a simplified case where $\phi_i=0$. Explicitly derive the solutions for the response of π_t, x_t, and i_t to shocks $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$. Additionally, explain the economic reasoning behind the responses.

3. General Equilibrium and Optimality in OLG (45 pts) Consider the two-age OLG model with utility of the form $U^t(c_t^t, c_{t+1}^t) = c_{t+1}^t$ and endowment process $e_t^t = G^t, e_{t+1}^t = 0$ where $G > 1$. That is, households only care about consumption when they are old and only receive an endowment of consumption goods when young, which is growing over time. As usual, time starts at $t = 0$, at which point there is a group of initial old who have $e_0^{-1} = 0$ consumption goods.

1. Define a sequential markets equilibrium where households buy a_t units of an Arrow Security when young at period t and receive $a_t(1 + r_{t+1})$ from those purchases when old in period $t + 1$.

2. Fully solve for the above equilibrium (ie, tell me what the equilibrium interest rate has to be in each period, along with the consumption and asset allocations).

3. List the Pareto Efficient allocation(s) for this economy.

4. Now suppose that each initial old household is endowed with a single handsome cat figurine in $t = 0$. The figurines contribute nothing to utility for anybody, but last forever. Define an equilibrium in which the figurines can be bought and sold and show that there is an equilibrium with a positive price. What is the growth rate of the price of handsome cat figurines?

5. Suppose that some people are worried about a handsome cat figurine *bubble* and a policy is implemented to make it illegal for the price of cat figurines to grow over time. What happens to consumption and welfare in the cat figurine price-ceiling economy relative to the laissez-faire economy as $t \to \infty$?

4. Optimal Fiscal Policy (45 pts) Consider an economy with a pollution externality. That is, for every unit of output there is some amount of pollution generated, call it $\Phi_t = \phi Y_t$. The production technology is neo-classical with capital and labor as inputs and a constant Solow Residual. Capital is accumulated according to the standard law of motion, and households have life-time utility given by:

$$U = \sum_{t=0}^{\infty} \beta^t [u(c_t, \ell_t) - \Phi_t]$$

Where $\ell_t \in [0, 1]$ is leisure. The government must finance an exogenous sequence of government expenditures, $(g_t)_{t=0}^{\infty}$.

1. Characterize the set of Pareto Efficient allocations in this economy.
2. Define and characterize the Tax-Distorted Competitive Equilibrium in this economy when proportional taxes are levied on labor and capital income. Households should take all prices, taxes, and aggregate variables as exogenous.
3. Derive the Implementability Condition for this economy and set up the Ramsey Planner's problem.
4. What happens to the capital income tax rate as $t \to \infty$?
5. Suppose that the Ramsey Planner had access to lump-sum taxes. Would the Ramsey Planner use only lump sum taxes? Why or why not.