The University of Texas at Austin
Department of Economics

MICROECONOMICS
Comprehensive Examination
July 2021

INSTRUCTIONS:

1. Please answer each of the four questions on separate pieces of paper.
2. Please write only on one side of a sheet of paper.
3. Please write in pen only.
4. When finished, please arrange your answers in the order in which they appeared in the questions, i.e. 1(a), 1(b), etc.

Question 1: Consider a perfectly competitive firms that faces a vector of input prices $\mathbf{w}$ and wishes to produce output y.

(a) Carefully define the firm's cost minimization problem. What is the program's value function? What is the program's solution?

(b) Assume that the production function $f(x)$ (where x is a vector of inputs) is continuous and strictly increasing in its arguments. List and prove carefully all the properties of the value function of the cost minimization problem. (Hint: there are 7 such properties).

Question 2:

(a) State what it means for a choice function to satisfy the Weak axiom of revealed preference.

(b) State what it means for a choice function to satisfy the Strong axiom of revealed preference.

(c) Suppose there are only two goods, the price vector is p and income I, and that a consumer's choice function $x(p, I)$ satisfies budget balancedness, $p \cdot x(p, I) = I, \forall (p, I)$. Show the following:

   i. Define the Slutsky matrix. If $x(p, I)$ is homogeneous of degree zero in (p, I), then the Slutsky matrix associated with $x(p, I)$ is symmetric.

   ii. If $x(p, I)$ satisfies WARP, then the 'revealed preferred to' relation, R, has no intransitive cycles (basically R viewed as a binary relation violates transitivity). Recall, that by definition, $x_1 R x_2$ if and only if bundle x_1 is revealed preferred to x_2.

Question 3

Two players have to divide a perfectly divisible cake of size 2 units. Half of the cake is made of chocolate, while the other half is made of marzipan. If a piece of cake contains x units of chocolate cake and y units of marzipan, player 1 gets utility $u_1(x, y) = x + y$ from this piece, while player 2 gets utility $u_2(x, y) = 2x + y$ from this piece. That is, player 1 cares only about the total size of the piece, while player 2 values chocolate twice as much as marzipan.

The players use the following "divide-and-choose" protocol, which has two stages. At the "divide" stage, one player divides the cake into two pieces. Then, at the "choose" stage, the other player chooses one of the two pieces for herself. More precisely, the divider chooses shares of chocolate and marzipan, $0 \leq x \leq 1$ and $0 \leq y \leq 1$, so as to create two pieces of cake, denoted (x, y) and $(1 - x, 1 - y)$. The chooser may choose either the piece (x, y) or the piece $(1 - x, 1 - y)$.

(a) Solve for the subgame perfect equilibrium, using backwards induction, when player 2 divides.

(b) Solve for the subgame perfect equilibrium, using backwards induction, when player 1 divides.

(c) Does a player prefer to be the divider or the chooser? Explain why, briefly.

Question 4

Consider a competitive labour market with a continuum of workers. A worker's productivity equals her type θ, where θ is drawn from the uniform distribution on the interval $[1, 3]$. The reservation wage of a worker of type θ is $r(\theta) = \alpha\theta$, where $\alpha \in (0, 1)$ is a constant parameter. A worker's type is only observed by her.

(a) Suppose that $\alpha < 0.5$. Solve for the competitive equilibrium with full employment. What is the critical value of α such that there exists an equilibrium with full employment?

(b) Suppose that $\alpha = 0.8$. Solve for a competitive equilibrium. Explain whether this equilibrium is (Pareto) efficient.

(c) Suppose now that a worker can choose an education level e, where the cost of education $c(e, \theta) = \frac{e}{\theta}$. If a worker with education e and type θ accepts a job at a wage of w, her payoff equals $w - c(e, \theta)$. If she stays unemployed, her payoff is $r(\theta) - c(e, \theta)$. Solve for a partially separating Perfect Bayesian equilibrium, where behavior is as follows: each worker whose type is in the set $[1, \hat{\theta})$ chooses $e = 0$; each worker whose type is in the set $[\hat{\theta}, 3]$ chooses education level $e^* > 0$.

   i. Solve for the wages that must be paid at $e = 0$ and $e = e^*$, as a function of $\hat{\theta}$.

   ii. Using the incentive constraint on the low productivity workers, derive the minimum value of e^*.

   iii. Specify fully the equilibrium strategies and beliefs when $\hat{\theta} = 2$.