Presupposition and Partiality: Back to the Future?

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Abstract. In this paper it is shown how a partial semantics for presuppositions can be given which is empirically more satisfactory than its predecessors, and how this semantics can be integrated with a technically sound, compositional grammar in the Montagovian fashion. Additionally, it is argued that the classical objection to partial accounts of presupposition, namely that they lack 'flexibility', is based on a misconception. Partial logics can give rise to flexible predictions without postulating any ad hoc ambiguities. Finally, it is shown how the partial foundation can be combined with a dynamic system of common ground maintenance to account for accommodation.

Keywords: presupposition projection, partial logic, type theory, Montague Grammar, flexibility, accommodation

1. Introduction

The use of partiality for the treatment of presupposition predates all other approaches: when Frege (1892) introduced the notion of presupposition he tied it explicitly to the possibility of sentences lacking a truth value. Since Strawson's rediscovery of presupposition and its partial treatment, an enormous number of partial and multivalent accounts have been proposed, for example Strawson (1950), Van Fraassen (1971), Herzberger (1973), Keenan (1973), Blau (1978), Martin (1979), Thomason (1979), Seuren (1980), Bergmann (1981), Humberstone (1981), Link (1986) and Burton-Roberts (1989). One might then think that the subject has been done to death. But the current paper should make it clear that, on the contrary, important issues in the partial treatment of presupposition have up until now not been studied adequately, and significant lines of research have never been given a technical explication.

From a technical point of view, partial semantics offers the attractive possibility of basing an account of presupposition in a well understood logical setting, and one where the basic intuition (that sentences may fail to be true or false) is quite clear. As will be shown, the ease with which logical properties of the system can be assessed in turn facilitates exploration of the linguistic predictions of the theory. But the theory itself must include more than just a propositional or even first order logic, such as are typically studied by logicians interested in partiality and multivalence. The theory must also explicate the relation between sentences of natural language and sentences of the underlying propositional/first order system, and clarify what the place of the partial semantics is within the interpretation process.

Although the literature on partial approaches to presupposition has been largely concerned with propositional systems, there have been some proposals for partial and multivalent systems in which the interaction of presuppositions and quantifiers have been studied: we are thinking here of Karttunen & Peters (1979), and proposals by Cooper (1983) and Hausser (1976). We do not wish to deny that there are many important insights in this earlier work, but there do remain problems. Karttunen and Peters' proposal suffers from empirical problems since although presupposition expressions may occur in the scope of quantifiers, the presuppositions themselves may not be bound in their system. Cooper provided a solution to this problem, but in his proposal the presuppositions associated with quantifiers must be stipulated for each quantifier, and do not have any independent empirical or technical motivation. Hausser's proposals, first put forward in the earliest days of Montague Grammar, suffer from technical shortcomings: the partial type theory he uses is a special purpose formalism which does not appear to maintain the attractive logical properties of classical type theory, and it is difficult to establish formally that derivations in his proposed grammar proceed with the desired results. In the process of presenting our own account, which contains a number of substantially new ingredients, we also hope to succeed in showing how the results achieved by earlier researchers can be tidied up. This will clarify which aspects of the earlier accounts represent real obstacles to progress, and which are better seen as mere technical shortcomings.

One empirical issue in particular will be highlighted in this paper: the interaction of presupposition and quantification. This may be illustrated with the following examples:

(1) Somebody managed to succeed George V on the throne of England.

(2) A fat man pushes his bicycle.

(3) Everyone who serves his king will be rewarded.

(4) Every nation cherishes its king.

These examples are drawn from Karttunen & Peters (1979) and Heim (1983). In the fourth example, for instance, the presupposition trigger "its king", carrying the presupposition that the referent of "its" has a king, occurs in the scope of the quantifier "every". The possibility of what might be called "quantifying in to presuppositions" was little studied before these papers.[1]

The main goals of this paper are to show how a partial semantics for presupposition can be given which is empirically more satisfactory than its predecessors, and to demonstrate that this semantics can be integrated within a technically clean, compositional grammar, in the spirit of Montague (and also Frege, it might be said). To this end, we will utilise recent formal developments, primarily Muskens (1989)'s partial type theory. It has been argued that a partial approach to presuppositions is doomed to failure, since it lacks the required flexibility (see e.g., van der Sandt (1989) and Soames (1979)). However, we shall argue that the examples which are standardly presented as problematic for the partial or multivalent approach do not provide a *knock down* argument for this position. In fact, we show that it is possible for an account of presupposition in terms of partial logic to make flexible predictions about projection, and, additionally, that this account can be combined with a dynamic approach to common ground maintenance to model accommodation. In general, we point to several promising lines of future research concerning the effect of pragmatic factors on the partial interpretation of utterances.

The remainder of this paper is organized as follows: we start at the basics by discussing various partial version of propositional logic and reviewing their applicability to presuppositions in section 2. In section 3 we then turn to Muskens' partial interpretation of type theory, which is used in section 4 as the representation language of a Montagovian grammar which includes presuppositions. In section 5 we discuss some of the traditional objections raised against accounts of presupposition based on partial logic, and describe two extensions of the partial account which meet these objections; in section 5.2 we sketch how a theory can be developed in terms of partial logic which gives rise to 'flexible' projection predictions, while in section 5.3 it is illustrated how a partial account of presupposition can be combined with a dynamic model of common ground maintenance allowing presupposition accommodation.

2. Partial Propositional Logic

We must start at a low-level, re-introducing a number of important concepts in the partial approach to presuppositions. In this section we discuss alternative interpretations of the language of propositional logic defined over a set of propositional constants $\mathbb{P}$, together with one extra operator to be defined below. In a sense, the most basic partial logic is the one from Kleene (1952),

[1] Note that the question of what happens when a presuppositional expression occurs bound within the scope of a quantifier is not to be confused with the issue of what presuppositions (e.g., existence of a non-trivial quantificational domain) are triggered by quantifiers themselves.

which is generally known as *strong Kleene*. In terms of this *mother of partial logics* a number of well-known partial logics can be defined (see e.g., Thijsse (1992)). In strong Kleene a propositional formula can be either true (and not false), false (and not true) or neither (true nor false). Following Belnap (1979) we refer to these three, so-called *truth-combinations* as T(rue), F(alse) and N(either) respectively. The following truth-tables capture the strong Kleene interpretation of the basic propositional language.

DEFINITION 1 (Strong Kleene)

$\wedge$	T	F	N
T	T	F	N
F	F	F	F
N	N	F	N

$\rightarrow$	T	F	N
T	T	F	N
F	T	T	T
N	T	N	N

$\vee$	T	F	N
T	T	T	T
F	T	F	N
N	T	N	N

	$\neg$
T	F
F	T
N	N

If we want to say something about presuppositions in this set-up two things are needed: (i) we need to know where presuppositions arise and (ii) we need to know when one formula is a presupposition of another formula. To achieve the first we add a binary presupposition-operator to the language, Blamey's *transplication*. Define:

DEFINITION 2
If φ, π are formulae, then $\varphi_{\langle\pi\rangle}$ is a formula.

The intuition behind this construction is that π is an *elementary presupposition* associated with φ.[2] One way to look at elementary presuppositions is as presuppositions which are triggered in the lexicon. For example, a word like *regret* comes with an elementary presupposition to the effect that the proposition which is regretted is true. Consider:

(5) Bill regrets that Mary is sad.

This sentence is represented schematically by a formula of the form $q_{\langle p\rangle}$ where p represents the proposition that Mary is sad, and q the proposition that Bill regrets this. In general, the interpretation of $\varphi_{\langle\pi\rangle}$ can be characterized as follows: $\varphi_{\langle\pi\rangle}$ is True iff both π and φ are True, and $\varphi_{\langle\pi\rangle}$ is False iff π is True and φ is False. This gives rise to the following truth-table:

[2] The 'elementary presupposition' terminology and the subscript notation derive from van der Sandt (1989). Blamey uses π/φ as notation for transplication.

DEFINITION 3 (Elementary Presuppositions)

	T	F	N
T	T	N	N
F	F	N	N
N	N	N	N

There is an alternative to this binary presupposition operator, namely the introduction of a *unary* presupposition operator. Such an operator, call it $\partial\pi$ (intuition: π is presupposed), might have the following truth-table:

DEFINITION 4 (Unary Presupposition Operator)

	∂
T	T
F	N
N	N

In fact, ∂ is the static version of Beaver's presupposition operator, used for the first time in Beaver (1992). The reader is invited to check that $(\partial\pi \wedge \varphi) \vee (\partial\pi \vee \neg\partial\pi)$ has exactly the truth-table of $\varphi_{\langle\pi\rangle}$.

Now we come to the question of when an arbitrary formula φ presupposes some formula π.

DEFINITION 5 (Presuppose)
φ presupposes π iff whenever π is not True, φ is Neither true nor false.

This definition is in the Strawsonian spirit; after all, when the presupposition (π) is not satisfied (that is: not True), the sentence (φ) as a whole does not make sense: it is Neither true nor false. We can also put it as follows: φ presupposes π iff whenever φ is defined (either True or False), π is True. Going one step further, we can speak of the *maximal* presupposition of φ, which is the logically strongest proposition presupposed by φ. Given that φ presupposes π iff $\varphi \vee \neg\varphi \models \pi$ (where $\models$ is defined as preservation of Truth), it follows that the maximal presupposition may be easily identifiable. (See for example Karttunen & Peters (1979) or Cooper (1983)). By convention, by *the* presupposition of φ, we mean the maximal presupposition, given by the disjunction of truth and falsity conditions of φ. We can equate the maximal presupposition of φ with $\varphi \vee \neg\varphi$, but observe that this formula is likely to contain elementary presuppositions itself. There are various systematic ways to find a formula which is itself devoid of elementary presuppositions but which is nevertheless equivalent with $\varphi \vee \neg\varphi$. For example: Kracht (1994) defines an algorithm for *presuppositional normal forms*. Another method which is often used is based on translations into classical logic. Here we follow the latter method. Appendix B contains two functions from partial into classical logic: TR^+ and TR^-. $\mathsf{TR}^+(\varphi)$ produces a classical formula which is true whenever φ is True, and $\mathsf{TR}^-(\varphi)$ yields a classical formula which is true whenever φ is False. Now the (maximal) presupposition of φ, designated as $\mathsf{PR}(\varphi)$, is defined as $\mathsf{TR}^+(\varphi) \vee \mathsf{TR}^-(\varphi)$. So what is the presupposition of $\varphi_{\langle\pi\rangle}$? Some easy calculations will show that

$$\mathsf{PR}(\varphi_{\langle\pi\rangle}) \Leftrightarrow \pi \wedge \mathsf{PR}(\varphi).$$

In other words, $\varphi_{\langle\pi\rangle}$ (at least) presupposes π, as intended. But how about other formulae? A well-known feature of elementary presuppositions is that sometimes they survive when embedded under one or more logical operators, while at other times they do not. The problem of predicting when which presuppositions survive is known as the *projection problem,* a phrase suggested in Langendoen & Savin (1971). Consider the following natural language examples:

(6) It is not true that Bill regrets that Mary is sad.

(7) If Bill regrets that Mary is sad, then he'll soothe her.

In an intuitive sense both these sentences seem to presuppose that Mary is sad. It is easily seen that $\mathsf{PR}(\neg\varphi) \Leftrightarrow \mathsf{PR}(\varphi)$. Thus, translating (6) in a strong Kleene based representation language yields the prediction that it shares its presupposition with (5). But what about (7)? Here is the general rule:

$$\mathsf{PR}(\varphi \rightarrow \psi) \Leftrightarrow (\mathsf{PR}(\varphi) \vee \mathsf{TR}^+(\psi)) \wedge (\mathsf{TR}^+(\neg\varphi) \vee \mathsf{PR}(\psi))$$

We can represent (7) as $(q_{\langle p\rangle} \rightarrow r)$, where p is the proposition Mary is sad, q the proposition that Bill regrets this and r the proposition that Bill soothes Mary.[3] Given what we have seen so far this means that the following presupposition is (incorrectly) predicted for (7).

$$\mathsf{PR}(q_{\langle p\rangle} \rightarrow r) \Leftrightarrow p \vee r$$

In words, the predicted presupposition can be paraphrased as "either Mary is sad, or Bill will soothe her".

Above we noted that it is possible to define a number of interesting logics *in terms of* strong Kleene. Here we briefly mention two of those: Peters (1975) three-valued logic (called middle Kleene in Krahmer (1994)) and weak Kleene (a.k.a. as Bochvar's internal logic, viz. Bochvar (1939)). To begin with the former, the Peters connectives are given by the following truth-tables (to distinguish them from the strong Kleene ones we add a dot above them).

[3] Throughout this paper we tacitly assume that the only source of partiality comes from failing presuppositions.

DEFINITION 6 (Middle Kleene)

$\dot{\wedge}$	T	F	N
T	T	F	N
F	F	F	F
N	N	N	N

$\dot{\rightarrow}$	T	F	N
T	T	F	N
F	T	T	T
N	N	N	N

$\dot{\vee}$	T	F	N
T	T	T	T
F	T	F	N
N	N	N	N

	$\neg$
T	F
F	T
N	N

These can be defined in terms of the strong Kleene system as follows:

DEFINITION 7 (Middle Kleene connectives in terms of strong Kleene)

1. $\varphi \mathbin{\dot{\wedge}} \psi \ = (\varphi \wedge \psi) \vee (\varphi \wedge \neg\varphi)$
2. $\varphi \mathbin{\dot{\vee}} \psi \ = (\varphi \vee \psi) \wedge (\varphi \vee \neg\varphi)$
3. $\varphi \mathbin{\dot{\rightarrow}} \psi = (\varphi \rightarrow \psi) \wedge (\varphi \vee \neg\varphi)$

The intuition behind these definitions can be put as follows: the leftmost subformula has to be defined before the rightmost subformula becomes relevant. If we represent example (7) as $(q_{\langle p \rangle} \mathbin{\dot{\rightarrow}} r)$, we get different predictions about presupposition projection. Some calculations show that the general rule is

$$\mathsf{PR}(\varphi \mathbin{\dot{\rightarrow}} \psi) \Leftrightarrow \mathsf{PR}(\varphi) \wedge (\mathsf{TR}^{+}(\neg\varphi) \vee \mathsf{PR}(\psi)).$$

For our example this means that it is (correctly) predicted that (7) presupposes that Mary is sad.

We finally mention the (internal) Bochvar/weak Kleene alternative. It differs from the other two Kleene systems discussed above in that it has a different underlying intuition for the N value. In strong Kleene it is understood as Neither true nor false, while in weak Kleene it is better thought of as *Nonsense*. The truth-tables are set-up according to the principle that when a subformula does not make sense, then the entire formula is nonsensical.

DEFINITION 8 (Weak Kleene)

$\ddot{\wedge}$	T	F	N
T	T	F	N
F	F	F	N
N	N	N	N

$\ddot{\rightarrow}$	T	F	N
T	T	F	N
F	T	T	N
N	N	N	N

$\ddot{\vee}$	T	F	N
T	T	T	N
F	T	F	N
N	N	N	N

	$\neg$
T	F
F	T
N	N

Even though the underlying philosophy of the N value is different, it is possible to define the weak Kleene connectives in terms of the strong Kleene ones. To separate the weak Kleene connectives from the others, we place two dots above them.

DEFINITION 9 (Weak Kleene connectives in terms of strong Kleene)

1. $\varphi \mathbin{\ddot{\wedge}} \psi = (\varphi \wedge \psi) \vee (\varphi \wedge \neg\varphi) \vee (\psi \wedge \neg\psi)$
2. $\varphi \mathbin{\ddot{\vee}} \psi = (\varphi \vee \psi) \wedge (\varphi \vee \neg\varphi) \wedge (\psi \vee \neg\psi)$
3. $\varphi \mathbin{\ddot{\rightarrow}} \psi = (\varphi \rightarrow \psi) \wedge (\varphi \vee \neg\varphi) \wedge (\psi \vee \neg\psi)$

Weak Kleene makes clear and uniform predictions concerning projection: every elementary presupposition projects, no matter where it originated. In the linguistic literature this is known as the *cumulative* analysis of presuppositions (originally introduced by Langendoen & Savin (1971)). Here is one instance:

$$\mathsf{PR}(\varphi \mathbin{\ddot{\rightarrow}} \psi) \Leftrightarrow (\mathsf{PR}(\varphi) \wedge \mathsf{PR}(\psi)).$$

It is not difficult to come up with counterexamples to this prediction. Consider:

(8) If Mary is sad, then Bill regrets that Mary is sad.

This sentence does not carry an intuitive presupposition to the effect that Mary is sad, nevertheless translating the implication using $\ddot{\rightarrow}$ would predict exactly that. Still, weak Kleene can be useful in a theory of presuppositions as we shall see below.

Before we turn to Type Theory let us give the formal definition of strong Kleene propositional logic (PL). We follow the –rather compact– format of Muskens (1989:42).[4] We have a distributive lattice over $\{$T,F,N$\}$, in which the meet $\cap$ corresponds with conjunction, the join $\cup$ with disjunction and the complement $-$ with negation . This gives rise to the following Hasse diagram, called L3 (cf. Belnap (1979)).

$$\begin{array}{c} \mathrm{T} \\ \uparrow \\ \mathrm{N} \\ \uparrow \\ \mathrm{F} \end{array}$$

For instance, to find the value of a conjunction of T and N, we look at T $\cap$ N, the highest element which is at least as low in the ordering as both T and N (given the properties of L3 this amounts to the lowest of the two), which is N. On the other hand the disjunction of T and N, T $\cup$ N should be the lowest element at least as high as both T and N, which is T. Now let $V : I\!\!P \rightarrow \{T,F,N\}$ be some valuation function. Define $[\![\varphi]\!]_V$ (the interpretation of φ under V):

[4] The main difference is that Muskens discusses four-valued interpretations, while we restrict our attention to the three valued ones. We would like to point out that the main points we want to make in this paper are independent of this choice. Indeed it could be argued that the four-valued alternative would provide a more natural starting point for discussing two-dimensional approaches to presuppositions such as Herzberger (1973), Karttunen & Peters (1979) or Cooper (1983).

DEFINITION 10 (Strong Kleene PL with transplication)

1.	$[\![p]\!]_V$	$=$	$V(p)$, iff $p \in I\!P$
2.	$[\![\neg\varphi]\!]_V$	$=$	$-[\![\varphi]\!]_V$
3.	$[\![\varphi \wedge \psi]\!]_V$	$=$	$[\![\varphi]\!]_V \cap [\![\psi]\!]_V$
4.	$[\![\varphi_{\langle\pi\rangle}]\!]_V$	$=$	T, iff $[\![\pi]\!]_V =$ T and $[\![\varphi]\!]_V =$ T
	$[\![\varphi_{\langle\pi\rangle}]\!]_V$	$=$	F, iff $[\![\pi]\!]_V =$ T and $[\![\varphi]\!]_V =$ F

$\varphi \vee \psi$ is defined as $\neg(\neg\varphi \wedge \neg\psi)$ and $\varphi \rightarrow \psi$ is defined as $\neg\varphi \vee \psi$.

3. Partiality and Type Theory

A characteristic feature of *Montague Grammar*[5] is that natural language expressions are translated into a representation language called *Intensional Logic* (IL). Several authors have argued for a replacement of IL by *Two-sorted Type Theory* (TY_2) (see for example Gallin (1975), Groenendijk & Stokhof (1984) and, of course, Muskens (1989)). TY_2 is essentially the logic of Church (1940) —based on the type-theoretical work of Russell and Ramsey early this century— but with an extra ground type s. The system we discuss here is TY_2^3 (*Three-valued, two-sorted type theory*) from Muskens (1989).

DEFINITION 11 (TY_2^3 types)

1. e, s and t are types,
2. if α and β are types, then $(\alpha\beta)$ is a type.

So there are two sorts of types: basic types (including s, which is not basic in IL) and complex types. The TY_2^3 expressions are defined in the following fashion. Assume that we have a set CON_α of constants of type α, and VAR_α of variables of type α. An expression of type t is called a formula.

DEFINITION 12 (TY_2^3 syntax)

1. If φ and ψ are formulae, then $\neg\varphi$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$ and $(\varphi \wedge \psi)$ are formulae.
2. If φ is a formula and $x \in \mathsf{VAR}$, then $\forall x\varphi$ and $\exists x\varphi$ are formulae.
3. If A is an expression of type $(\alpha\beta)$ and B is an expression of type α, then (AB) is an expression of type β.

[5] We use the term Montague Grammar to refer to the so-called PTQ fragment as it was described in Montague (1974). Good introductions to Montague Grammar are Dowty, Wall & Peters (1981) and Gamut (1991).

4. If A is an expression of type β and $x \in \mathsf{VAR}_\alpha$, then $\lambda x(A)$ is an expression of type $(\alpha\beta)$.

5. If A and B are expressions of the same type, then $(A \equiv B)$ is a formula.

6. $\star$ is a formula.

The main difference with the syntax of IL (for those in the know) is the absence of the notorious caps and cups.

Let us now turn to the semantics. TY_2^3 models are defined as follows: $M = \langle \{D_\alpha\}_\alpha, I\rangle$. Here $\{D_\alpha\}_\alpha$ is a TY_2^3 frame, in which each type α is associated with its own domain D_α in such a way that D_e and D_s are non-empty sets, $D_t = \{\text{T,F,N}\}$ (*this is where the partiality comes into play*) and $D_{(\alpha\beta)}$ is the set of (total) functions from D_α to D_β. I is the interpretation function of M. It has the set of constants as its domain such that $I(c) \in D_\alpha$ for all $c \in \mathsf{CON}_\alpha$. We also have a set of total assignments G such that for any $g \in G$ and $x \in \mathsf{VAR}_\alpha$, $g(x) \in D_\alpha$. $g[d/x]$ is the assignment which differs only from g at most in that $g[d/x](x) = d$. Define $[\![A]\!]_{M,g}^{\mathrm{TY}_2^3}$ (the interpretation of a TY_2^3 expression A in a model M with respect to an assignment g, suppressing sub- and superscripts where this can be done without creating confusion):

DEFINITION 13 (TY_2^3 semantics)

1. $[\![c]\!] = I(c)$, if $c \in \mathsf{CON}$
   $[\![x]\!] = g(x)$, if $x \in \mathsf{VAR}$
2. $[\![\neg\varphi]\!] = -[\![\varphi]\!]$
3. $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$
4. $[\![\forall x_\alpha \varphi]\!]_g = \bigcap_{d \in D_\alpha} [\![\varphi]\!]_{g[d/x]}$
5. $[\![AB]\!] = [\![A]\!]([\![B]\!])$
6. $[\![\lambda x_\alpha A]\!]_g =$ the function f such that $f(d) = [\![A]\!]_{g[d/x]}$, for all $d \in D_\alpha$
7. $[\![A \equiv B]\!] =$ T, iff $[\![A]\!] = [\![B]\!]$
   $=$ F, iff $[\![A]\!] \neq [\![B]\!]$
8. $[\![\star]\!] =$ N

Here $-, \cap$ and $\bigcap$ are again operations on L3. Disjunction, implication and existential quantification are defined in the normal fashion. This is a very-well behaved logic, with lots of nice meta-theoretical results.[6] Notice that clauses

[6] Cf. Muskens (1989:ch. 5). For instance, the system has a sound and complete axiomatization with respect to a class of generalized frames, and — unlike IL — it enjoys Church-Rosser (or diamond) normalization.

2 and 3 are the same as for propositional logic. Hence, TY_2^3 follows the strong Kleene pattern. We use the following abbreviations:

DEFINITION 14 (Abbreviations)

$\top$	abbreviates	$\star \equiv \star$
$\varphi \vee \psi$	abbreviates	$\neg(\neg\varphi \wedge \neg\psi)$
$\varphi \rightarrow \psi$	abbreviates	$\neg(\varphi \wedge \neg\psi)$
$\exists x\varphi$	abbreviates	$\neg\forall x\neg\varphi$
$\varphi \dot{\wedge} \psi$	abbreviates	$(\varphi \wedge \psi) \vee (\neg\varphi \wedge \varphi)$
$\varphi \ddot{\wedge} \psi$	abbreviates	$(\varphi \wedge \psi) \vee (\neg\varphi \wedge \varphi) \vee (\neg\psi \wedge \psi)$
$\partial\pi$	abbreviates	$(\pi \equiv \top) \vee \star$
$\varphi_{\langle\pi\rangle}$	abbreviates	$\partial\pi \dot{\wedge} \varphi$

Finally, we observe that TY_2^3 supports the following facts:[7]

FACT 1 (Equivalences)

1. $\lambda v(\varphi)\beta$ is equivalent with $[\beta/v]\varphi$, provided β is free for v in φ.
2. $\lambda v(\varphi\ v)$ is equivalent with φ, provided v doesn't occur free in φ.
3. $\lambda v\varphi$ is equivalent with $\lambda w[w/v]\varphi$, provided β is free for v in φ.

The first of these facts is known as either lambda-conversion or beta-reduction, the second as eta-conversion, and the third as alpha-conversion.

4. Presuppositional Montague Grammar: worked examples

Let us now turn to a reformulation of Karttunen & Peters' system using TY_2^3. We shall refer to the resulting system as *Presuppositional Montague Grammar*. Before we start, a note about the notation. Application is written without brackets, on the understanding that association is to the left. So, *soothe* abi should be read as in state i, b (subject) soothes a (object).[8] Notice that this is analogous to the IL formula $^\vee$*soothe* ab, IL instances of cup operators being replaced by explicit function application to a state variable i. Similarly, the IL formula $^\wedge$*soothe* ab corresponds with $\lambda i($*soothe* $abi)$ in TY_2, IL's cap operators being replaced with explicit abstraction over states (compare the embedding of IL in TY_2, Gallin (1975)). The fragment we present is intensional: sentences of English are translated into TY_2^3 expressions of type (st),

[7] Where $[\beta/v]\varphi$ is the substitution of β for all free occurrences of v in φ. We say that a variable y is free for v in φ iff no free occurrence of v in φ is within the scope of a quantifier $\exists y$ or $\forall y$ or a lambda-operator λy.

[8] In general, $\xi\varphi_1 \ldots \varphi_n$ should be read as $(\ldots(\xi\varphi_1)\ldots\varphi_n)$.

such expressions will be called *propositions*. Otherwise, we introduce the relevant concepts as we go along. Let us begin with example (1), repeated here as (9). Following Karttunen and Peters, we assume that the English phrase "manage to X" carries a presupposition (*conventional implicature* in Karttunen and Peters' terms) that the subject had difficulty in *X*-ing, and for this purpose we use a constant *difficult* in the translation. Note that we do not especially wish to push this as an analysis of "manage", but use it to exemplify the difference between Karttunen and Peters' system and the present one — the relevant differences could equally well be discerned with more paradigmatic examples of presuppositions, such as those arising with definite descriptions or factives.

(9) Somebody managed to succeed George V (on the throne of England).

The problem with this sentence is that it has an odd flavour. Karttunen & Peters' system does not account for this oddity. Their system predicts that (9) presupposes that some individual, about whom no more is known, had difficulty to succeed George V. The problem is that this presupposition is too weak. For, as Karttunen and Peters themselves observed, it is certainly the case that there are people for whom the operation of succeeding George V is/was difficult, so that the weak presupposition predicted in their system is satisfied; (9) does not suffer from presupposition failure, it makes perfect sense.

Let us now look at example (9) from the perspective of this paper. Here is the classical PTQ structure derived for this example:[9]

(10) $[$ somebody $[$ managed to $[$ succeed George V $]^5]^8]^4$

The superscripts 4, 5 and 8 refer to three of the seventeen rules of which the PTQ fragment consist. Essentially, all three are straightforward rules of *Functional Application* (FA). Appendix A contains all the relevant details about Presuppositional Montague Grammar. It describes how trees are constructed and defines a function $(.)^\bullet$ from trees to TY_2^3 expressions. Informally, functional application is translated as follows: $([\alpha\ \ \beta]_{\mathsf{fa}})\ =\ \alpha^\bullet\beta^\bullet$. We need the following lexical items:[10]

$$\begin{array}{ll} \text{somebody}^\bullet & = \lambda P \lambda i \exists x (P\ xi) \\ \text{managed to}^\bullet & = \lambda P \lambda x \lambda j (P\ xj_{\langle(\textit{difficult}\ P)\ xj\rangle}) \\ \text{succeed}^\bullet & = \lambda Q \lambda y (Q \lambda x (\textit{succeed}\ xy)) \\ \text{George V}^\bullet & = \lambda P (P\ g) \end{array}$$

[9] We shall ignore the PP *on the throne of England*.

[10] Here and elsewhere we use the following type-convention: p, q are variables ranging over propositions (have type (st)), P over properties (type $e(st)$), Q over quantifiers (type $(e(st))(st)$), x, y over individuals (type e), i, j over states (type s), g is a constant of type e, *succeed* is a constant of type $(e(e(st)))$ and *difficult* is a constant of type $((e(st))(e(st)))$.

Below $\Longrightarrow_\eta$ indicates that one or more eta-reductions have been carried out, and $\Longrightarrow_\lambda$ that that one or more lambda-reductions have been applied.

1. ([succeed George V])$^\bullet$ $=$
   succeed$^\bullet$ George V$^\bullet$ =
   $\lambda Q \lambda y(Q \lambda x(\textit{succeed}\ xy))\lambda P(P\ g) \Longrightarrow_\lambda$
   $\lambda y(\textit{succeed}\ gy) \Longrightarrow_\eta$
   $\textit{succeed}\ g$
2. ([manage to [succeed George V]])$^\bullet$ $=$
   $\lambda P \lambda x \lambda j(P\ xj_{\langle(\textit{difficult}\ P)\ xj})\textit{succeed}\ g \Longrightarrow_\lambda$
   $\lambda x \lambda j(\textit{succeed}\ gxj_{\langle(\textit{difficult}\ (\textit{succeed}\ g))\ xj})$
3. ([somebody [managed to [succeed George V]]])$^\bullet$ $=$
   $\lambda P \lambda i \exists y(P\ yi)\lambda x \lambda j(\textit{succeed}\ gxj_{\langle(\textit{difficult}\ (\textit{succeed}\ g))\ xj}) \Longrightarrow_\lambda$
   $\lambda i \exists y(\textit{succeed}\ gyi_{\langle(\textit{difficult}\ (\textit{succeed}\ g))\ yi\rangle})$

The derived proposition can be phrased as follows: it is a function from states to truth-values, and given a state s there has to be someone of which it is asserted that he succeeded George V in s and presupposed that *he* (and not just any person) had difficulty to succeed George V in s. What presupposition is predicted by the use of TY_2^3 for this sentence, and is it an improvement over the predictions derived in Karttunen and Peters' system?

First, let us define what it means for a type-theoretical formula to presuppose another.

DEFINITION 15 (Presuppose: TY_2^3)
Let π and φ be expressions of type st. We say that φ presupposes π iff for all models M, assignments g and states s:

$$[\![\pi\ s]\!]_{M,g} \neq \mathrm{T} \Rightarrow [\![\varphi\ s]\!]_{M,g} = \mathrm{N}.$$

When φ is of the form $\lambda i \psi$, where ψ is itself λ-free, we can define

$$\mathsf{PR}(\varphi) = \lambda j(\mathsf{TR}^+(\varphi\ j) \vee \mathsf{TR}^-(\varphi\ j)).$$

In the case of (9) this amounts to the following:

(11) $\lambda i(\exists y((\textit{difficult}\ (\textit{succeed}\ g))\ yi \wedge \textit{succeed}\ gyi) \vee$
$\forall y((\textit{difficult}\ (\textit{succeed}\ g))\ yi \wedge \neg \textit{succeed}\ gyi))$

In words: either there is someone who had difficulty succeeding George V but did so anyway or it was difficult for everyone to succeed George V and no-one actually did succeed him. Notice that the first disjunct gives the condition under which (9) is True, while the second disjunct gives the condition under which it is False. That is: these conditions tell us when —in the Strawsonian fashion— example (9) makes 'sense'.

The problem with the sentence in need of explanation is its oddity, does the presupposition we just derived capture this oddity? Notice that both disjuncts of the presupposition are contradicted by history, since George VI did not have a particularly hard time following up his predecessor as it was his birthright; the presupposition is false. So, intuitively (9) is predicted to be a case of presupposition failure, which may be taken as an explanation of its oddity. Does this outcome also correspond with the intuitions about sentence (9)? This is in fact a very difficult question. There does not seem to be any consensus as to what the intuitive presuppositions of (9) are. In fact, there is no consensus at all about presuppositions containing a free variable which is bound by a quantifier outside the scope of the presupposition. We believe that empirical research should clarify these matters (a first start in this direction is carried out in Beaver (1994)). Below we return to this issue.

Let us now turn to one of the examples from Heim (1983), say (4), repeated here as (12).

(12) Every nation cherishes its king.

We just mentioned that the intuitions about presupposition-quantification interaction differ widely. However, there appears to be consensus that presuppositions under universal quantifiers do not give rise to universal presuppositions. Thus, for instance, (12) does not come with an intuitive presupposition to the effect that *every nation has a king*. Nevertheless, this is the presupposition Heim's system predicts. Let us see how Presuppositional Montague Grammar does. To deal with this example in Montague Grammar we need to invoke the notorious rule of *quantifying-in* (QI, n labeled $14, n$ in the original PTQ fragment). Here is the schematic syntactic structure of (12), all rules follow the FA-pattern unless otherwise indicated:

$$[[\text{every nation}]\ [\text{he}_0\ [\text{cherishes}[his_0\ \text{king}]]]]_{\text{qi},0}$$

To fresh up your minds: the pronouns with a subscript are Montague's syntactic variables. The possessive his_0 is our addition.[11] Syntactically QI,0 replaces the first occurrence of *he*$_0$ for the NP *every nation*, and all subsequent syntactic pronouns with the same index are replaced for suitable anaphoric pronouns. The corresponding translation rule looks roughly as follows (again, the appendix contains the formal details).

$$([\alpha\ \beta]\text{qi}, n)^\bullet = \alpha^\bullet(\lambda x_n \beta^\bullet).$$

[11] The addition of his_n is not, strictly speaking, necessary. A first alternative is to analyze *his*$_n$ *king* as an abbreviation of *the king of he*$_n$. A second alternative is to isolate the meaning of *'s* and combine it with *he*$_n$ to form *his*$_n$. This second alternative is formally worked out in appendix A.

For this example we need the following translations of lexical items.[12]

every$^\bullet$ $= \lambda P_1 \lambda P_2 \lambda i \forall x(P_1\ xi \dot{\rightarrow} P_2\ xi)$
nation$^\bullet$ $=$ *nation*
he$_n{}^\bullet$ $= \lambda P(P\ x_n)$
cherishes$^\bullet$ $= \lambda Q \lambda y(Q \lambda x(\textit{cherish}\ xy))$
his$_n{}^\bullet$ $= \lambda P_1 \lambda P_2 \lambda i \exists y((P_1\ yi \dot{\wedge} \textit{of}\ yx_n i \dot{\wedge} P_2\ yx_n i)_{\langle \exists ! z(P_1\ zi \dot{\wedge} \textit{of}\ zx_n i)\rangle})$
king$^\bullet$ $=$ *king*

We have assumed that possessives trigger a uniqueness presupposition, but nothing hinges on that. Here are the crucial steps in the translation:

1. ([he$_0$ serves his$_0$ king])$^\bullet$ $=$
$\lambda i \exists y((\textit{king}\ yi \dot{\wedge} \textit{of}\ yx_0 i \dot{\wedge} \textit{cherish}\ yx_0 i)_{\langle \exists ! z(\textit{king}\ yi \dot{\wedge} \textit{of}\ yx_0 i)\rangle})$
2. ([every nation])$^\bullet$ $=$
$\lambda P \lambda j \forall z(\textit{nation}\ yj \rightarrow P\ yj)$
3. ([[every nation] [he$_0$ cherishes his$_0$ king]]$_{\mathrm{qi},0}$)$^\bullet$) $=$
$\lambda j \forall z(\textit{nation}\ zj \dot{\rightarrow} \exists y((\textit{king}\ yi \dot{\wedge} \textit{of}\ yzi \dot{\wedge} \textit{cherish}\ yzi)_{\langle \exists ! x(\textit{king}\ xi \dot{\wedge} \textit{of}\ xzi)\rangle}))$

Heim predicts that example (12) presupposes (13).

(13) Every nation has a king.

Things are different for the translation we just derived. It is easily seen that this formula does not give rise to a Heimian, universal presupposition. The presupposition of this formula amounts once again to the disjunction of the truth and falsity conditions, and as we have seen above the former is universal while the latter is existential. The predicted presupposition can be paraphrased as in (14).

(14) Either every nation has a king it cherishes or there is a nation which has a king it does not cherish.

In other words, a presupposition is predicted which is lot weaker than Heim's. Notice again, that the first disjunct paraphrases the condition under which (12) is True, while the second disjunct paraphrases the condition under which it is False; the disjunction tells us when (12) 'makes sense'.

The machinery we have used so far is essentially enough to deal with the remaining two Heimian examples from the introduction as well. Appendix

[12] For the sake of argument we have chosen to use the Peters/middle Kleene connectives in the translations, because they represent the Karttunen-style treatment of presuppositions. It should be stressed that for these examples nothing hinges in this choice. In fact, using strong Kleene connectives would lead to exactly the same predictions here.

A contains all the details which are involved to construct syntactic trees for these examples, and to calculate their corresponding TY_2^3 representation. As far as example (2) is concerned: the only non-alphabetical difference with (4) is that an *existential* quantifier is quantified-in and not a universal one. Since existential quantification is defined as the dual of universal quantification it is easily seen that no universal presupposition is predicted for this example either (contrary to Heim (1983) which predicts a universal presupposition for this example as well). Example (3) is slightly more involved since it contains a relative clause. However, these do not pose any problems for classical Montague Grammar (just use rule $2, n$). And again, no universal presupposition is predicted, but a weaker disjunctive one.

So what have we achieved so far? We have shown that it is possible to define a Montague Grammar which deals with both assertions and presuppositions, but does not run into the problems Karttunen & Peters' system has with sentences such as (1). What's more: the fragment can also deal with the quantificational examples discussed in Heim (1983) and which play an important rôle in the recent partial dynamic approaches to presuppositions. It is interesting to note that there is nothing dynamic about TY_2^3, it is just a standard static logic albeit a partial one. Actually, this is not the first attempt to partialize Montgaue Grammar in order to deal with presuppositions, Hausser (1976) and Cooper (1983) are two old (late seventies) predecessors. Our system is really in their spirit. The main difference is that we have benefited from the pioneering work of Muskens (1989), which arguably is the first 'clean' partialization of Montague Grammar with clear logical properties.

5. Extensions to the partial account

5.1. Alleged Limitations of the Partial Account

There are two types of objection which can be leveled at any theory of presupposition, that it predicts overly strong presuppositions, and that it predicts overly weak presuppositions. Both of these objections have been leveled at various aspects of multivalent accounts.

Of the objections regarding overly strong predictions the examples where a presupposition is *cancelled* are best known. An example is the following, where the speaker is clearly not committed to Bill being happy, despite the factive verb "know" being used:

(15) Mary doesn't *know* that Bill is happy, she merely believes it.

The standard solution to this problem within multivalent accounts is to postulate a second *presupposition cancelling* negation with a truth-table as the following:

DEFINITION 16 (Cancelling negation)

	$\sim$
T	F
F	T
N	T

However, this then meets with the further objection that linguistic evidence does not support the presence of a lexical ambiguity. It is argued that if negation is ambiguous in this way, we should expect there to be languages in which there are non-homophonous realisations of the two lexical entries, but Horn (1985) points out that whilst there are many languages with distinct negations, there is no language in which the distinction seems to correspond to the presupposition-projecting/presupposition-cancelling dichotomy. Not only is the postulation of a second negation *ad hoc*, but it is also a distinctly limited solution for what is clearly a wider problem. Sticking at least to cases of denial, observe that the following variant of the above example (from Beaver (1997a)) exhibits identical presupposition cancelling behaviour, at least when uttered by an Englishman:

(16) If Mary *knows* that Bill is happy, then I'm a Dutchman — she merely believes it.

It seems most undesirable for an ambiguity of implication to be postulated paralleling the claimed ambiguity of negation, solely so that this "non-standard" use of implication for denial can be treated.

The most troublesome of all the logical connectives with regard to presupposition is surely disjunction, a point made most forcefully by Soames (1979). Consider examples (17) - (23). The constuction *stop doing X* presupposes having done *X* before, while *start doing X* presupposes not having done *X* before. Projection of this presupposition can occur from the left disjunct (as in (17)), the right disjunct (18), or both disjuncts (19). Cancellation of the presupposition in the left disjunct can occur (20), as can cancellation of the presupposition in the right disjunct (21). Furthermore, simultaneous cancellation of presuppositions in both disjuncts can occur, either as a result of the assertions in each disjunct cancelling the presuppositions in the other (22), or as a result of the presuppositions of the disjuncts being inconsistent with each other (23).

(17) Either Bill has stopped smoking, or he doesn't have enough money to buy cigarettes.

(18) Either Bill doesn't have enough money to buy cigarettes, or he's stopped smoking.

(19) Either Bill has just stopped smoking, or else he's just started doing some exercise.

(20) Either Bill has just stopped smoking, or he never did smoke and just carried that lighter around as a pose.

(21) Either Bill always did smoke, but only when nobody was watching, or else he's just started smoking.

(22) Either Bill just stopped smoking, and never did drink, or else he just stopped drinking, and never did smoke.

(23) Either Bill has just stopped smoking, or else he's just started smoking.

It is quite impossible that any single multivalent truth table for disjunction will predict all these possibilities, and attempting to solve the problem by introducing a multiple lexical ambiguity for disjunction seems a most unattractive prospect.

As said, partial approaches to presuppositions have also been criticized for making predictions which are too weak. Within a multivalent semantics certain operators may behave as filters, neither allowing uniform projection of presuppositions nor forcing uniform cancellation. Various of the disjunctions that might be defined to meet one or other of the above examples are of this type. However, such filters do not work in quite the way that Karttunen envisaged when he coined the term *filter* in Karttunen (1973). For in the model he proposed there, presupposition triggers are thought of as being associated with a single presupposed proposition, "knows that Bill is happy", for instance, being associated with the proposition "Bill is happy". A filtering operator taking a sentence with this trigger as an argument could do either of two things: it could allow the proposition "Bill is happy" to be projected, or it could prevent that projection. But multivalent models exhibit more complex behaviour. Consider:

(24) If Mary is clever, she knows that Bill is happy.

Suppose that conditionals are given a semantics in accordance with the strong Kleene implication (or the middle Kleene/Peters one, for that matter). Then the sentence (24) does not simply carry the presupposition "Bill is happy". But neither does this presupposition vanish altogether. Rather we obtain a conditional presupposition to the effect "if Mary is clever then Bill is happy." As it happens, these conditional presuppositions are by now generally associated with Karttunen's work, since they do arise in Karttunen (1974) and Karttunen & Peters (1979) — see Geurts (1995) and Beaver (1997a) for discussion. Conditional presuppositions have been attacked, for instance in Gazdar (1979), as being inappropriately weak. Let us briefly point out here that there are also examples for which Karttunen-style conditional presuppositions do seem to capture the intuitions. Consider the following example from Beaver (1995).

(25) If Spaceman Spiff lands on Planet X, he will be bothered by the fact that his weight is higher than it would be on earth.

Here, the consequent is associated with the presupposition that Spaceman Spiff's weight is higher than it would be on earth. Intuitively we would not want to associate this presupposition with example (25) as a whole. Neither would we want it to disappear entirely. Rather, we would like to predict the conditional/filtered presupposition which strong or middle Kleene would predict and which may be paraphrased as "if Spaceman Spiff lands on Planet X his weight is higher than it would be on earth".

Thus, sometimes a presupposition which arises in the consequent of an implication should project strongly (as for (24)) while at other times a weaker presupposition is desired (viz. (25)). In fact, this is a good illustration of the fundamental point of criticism which has been leveled at partial and multivalent approaches to presuppositions: they lack the desired *flexibility*. Once a connective has been assigned a partial interpretation it makes rigid predictions concerning presupposition projection. Once implication is assigned a truth-table, any formula representing a conditional is associated with the same presupposition. Thus, given the truth-table of implication in definition 1 for strong Kleene only conditional presuppositions are predicted. By contrast, giving it a weak Kleene interpretation we never predict conditional presuppositions. Soames' examples clearly illustrate that projection from disjunctions is an even more flexible matter. However, giving disjunction a partial interpretation means that we always predict the same projection behavior.[13] So, the conclusion is that no single partial logic can account for all the projection facts, and this is indeed what is claimed in for instance van der Sandt (1989) and Soames (1979). However, this does not mean that the defender of a partial approach is forced to postulate multiple ambiguities, and it certainly does not mean that partial and multivalent logics are useless when it comes to the treatment of presupposition.

5.2. Flexibility: the Floating A Theory

There is an alternative to postulating a lexical ambiguity, dating back as far as Bochvar's original papers (Bochvar (1939), cf. also Frege's *horizontal* in Frege (1879)). Bochvar suggested that apart from the normal mode of assertion there was a second mode which we might term *meta-assertion*. The meta-

[13] Thus: strong Kleene disjunction yields uniform weak (conditional) presuppositions, middle Kleene disjunction predicts that elementary presuppositions from the left disjunct project, weak Kleene that every elementary presupposition projects.

assertion of φ, $A\varphi$, is the proposition that φ is true. This gives rise to the following truth-table:

DEFINITION 17 (Assertion operator)

	A
T	T
F	F
N	F

Thus $A\varphi$ is True iff φ is True, and False otherwise. Bochvar showed how within the combined system consisting of the internal connectives and this assertion operator a second set of *external* connectives could be defined: for instance the external conjunction of two formulae is just the internal conjunction of the meta-assertion of the two formulae ($\varphi \wedge_{\text{ext}} \psi =_{\text{def}} A\varphi \wedge_{\text{int}} A\psi$). For the present purposes A has a different use: it wipes out elementary presuppositions. Whatever is presupposed by some formula φ, it is easily seen that $A\varphi$ presupposes nothing. Here are some characteristic properties of A.

FACT 2 (A equivalences)

1. $A(\varphi_{\langle\pi\rangle})$ is equivalent with $A\pi \wedge A\varphi$
2. $A\partial\pi$ is equivalent with $A\pi$
3. $AA\varphi$ is equivalent with $A\varphi$
4. $A\varphi$ is equivalent with φ, if φ is defined
5. $\sim\varphi$ is equivalent with $\neg A\varphi$

The first and second equivalence say that the *presupposition wipe-out device* indeed wipes out presuppositions. The third equivalence illustrates that multiple A's have the same effect as a single one; you can't wipe-out presuppositions which are not there any more. The fourth equivalence is related to this: if φ is defined (always either True or False) and hence does not contain presuppositions, $A\varphi$ is equivalent with φ. The fifth and final equivalence shows that we can define the cancelling negation $\sim$ (Bochvar's external negation) in terms of $\neg$ and A.[14] Thus whilst the possibility of declaring natural language

[14] External negation, given that it can be defined as $\neg A(\varphi)$ where A is a sort of truth-operator, has often been taken to model the English paraphrases "it is not true that" and "it is not the case that". Although it may be that occurrence of these extraposed negations is high in cases of presupposition denial — we are not aware of any serious research on the empirical side of this matter — it is certainly neither the case that the construction is used in all instances of presupposition denial, nor that all uses of the construction prevent projection of embedded presuppositions. Thus the use of the term *external* for the weak negation operator, and the corresponding use of the term *internal* for the strong, is misleading, and does not reflect a well established link with different linguistic expressions of negation.

negation to be ambiguous between $\neg$ and $\sim$ exists within Bochvar's extended system, another possibility would be to translate natural language negation uniformly using $\neg$, but then allow that sometimes the proposition under the negation is itself clad in the meta-assertoric armour of the A-operator. But there is an additional advantage: there is no reason whatsoever to limit occurrences of A to propositions directly under the scope of negation. Why not let them float around freely?

There is no technical reason why the A operator should be restricted in its occurrence to propositions directly under a negation, and one could imagine developing a theory (call it the *floating-A theory*) where all occurrences of cancellation were explained away in terms of the occurrence of such an operator. The result would have the same logical possibilities open as in a system with an enormous multiplicity of connectives: for instance if the A operator could freely occur in any position around a disjunction, then the effects of having the following four disjunctions would be available: $\varphi \vee \psi$, $A\varphi \vee A\psi$, $A\varphi \vee \psi$ and $\varphi \vee A\psi$.

How can we employ the resulting flexibility in a floating A theory? For that, we need the following ingredients:

- each sentence is associated with a *set* of translations,
- over this set a *preference order* is defined, and
- the translations have to satisfy certain *constraints*.

These ingredients are also, at least conceptually, present in Link (1986), an intentionally idiosyncratic defense of partial logic in the analysis of presuppositions. Without particularly wanting to commit ourselves to a specific version of a floating A theory, let us look at one simple, possible interpretation of it.

We start from a single partial logic, say *weak Kleene*. First, we associate each sentence with a *set* of translations. Consider (a disambiguated syntactic analysis of) a sentence *S* and suppose that φ is an A-free weak Kleene-based expression representing *S*. The reader may think of φ as the TY_2^3 representation of *S* derived by (an extension of) the fragment sketched above. It should be stressed, however, that the floating A theory is not dependent on a Montagovian foundation; it works for any partial logic. The translation-set of *S* ($\mathsf{TS}(S)$) is the smallest set such that:

1. $\varphi \in \mathsf{TS}(S)$
2. Any formula η that results from replacing all occurrences of one or more formulae χ which are of the form $\psi_{\langle\pi\rangle}$ by $A\chi$ is an element of $\mathsf{TS}(S)$.

Second, we need to define a preference order over the translation set. How to do this? The intention is to keep the usage of the A operator as limited as possible; the default is that presuppositions project. We can interpret this as follows: if γ and δ are both elements of $\mathsf{TS}(S)$, then γ is preferred over δ (notation: $\gamma \prec \delta$) iff the number of A operators occurring in γ is lower then the number of A occurrences in δ.[15]

The result of defining the preference order in this way is that the prefered element of $\mathsf{TS}(S)$ will be the initial representation φ itself. When φ violates one of the constraints, a formula which is ordered below φ (and which, by definition, contains one or more occurences of the A operator) may turn out to be the most preferred one. And this immediately brings us to the third and final ingredient: the constraints. For now, we simple follow Stalnaker, van der Sandt and others and just require *consistency* and *informativity*. Informativity essentially says that no (sub-)formula should be redundant, consistency that no (sub-)formula should be inconsistent. The conditions can be defined completely analogous to the way van der Sandt's (1992) conditions are defined in Beaver (1997a:981). Thus, for instance, a TY_2^3 expression φ (of type st) is *inconsistent* iff there is no model M and state s such that φs is True in M. An expression φ is *not informative* iff it contains a subformula ψ such that for any model M and state s, whenever φs is True in M, then so is $\{\top/\psi\}\varphi s$ (where $\{\top/\psi\}\varphi$ is the expression derived from φ by substituting the occurrence of ψ with $\top$ (the tautological formula)). As in van der Sandt (1992), the assumption is that informativity and consistency apply at the level of (sub-)sentences. Now consider the following example.

(26) The king of France is *not* bald, since there is no king of France.

Schematically, this sentence is represented by an expression of the following form: $\lambda i(\neg(\varphi i_{\langle \pi i \rangle}) \mathbin{\ddot{\wedge}} \neg \pi i)$, where π expresses the proposition that there is a king of France and φ that the king of France is bald. The translation set of example (26) contains two representations:

(i) $\lambda i(\neg(\varphi i_{\langle \pi i \rangle}) \mathbin{\ddot{\wedge}} \neg \pi i)$
(ii) $\lambda i(\neg A(\varphi i_{\langle \pi i \rangle}) \mathbin{\ddot{\wedge}} \neg \pi i)$

The default reading of (26) is (i), and it is easily seen that it does not meet the consistency constraint: the second conjunct explicitly denies the presupposition of the first conjunct. Hence it is predicted that the second reading of (26), i.e., (ii), is the right one in this case. Consequently (26) is predicted to presuppose nothing, in particular not that there is a king of France. This indicates that we can replace the lexical ambiguity of negation which is common

[15] Needless to say this is a simplification, but it will do for the present purposes. It would be interesting to investigate the possibility of defining the ordering in terms of logical strength (e.g., using Blamey's double-barrelled entailment), but here we refrain from doing so.

in trivalent theories, by an essentially structural ambiguity, and in this respect it is not unlike the Russellian scope-based explanation of projection facts[16].

Let us now consider (23), repeated below as (27).

(27) Either Bill has just stopped smoking, or else he's just started smoking.

Schematically, this sentence is represented by a TY_2^3 expression of the form $\lambda i(\gamma i_{\langle \pi i\rangle} \ddot{\vee}\ \delta i_{\langle \neg\pi i\rangle})$, where π is the proposition that Bill has smoked before, γ the proposition that Bill has just stopped smoking and δ that he has just started smoking. The translation set of this example contains 4 TY_2^3 expressions:

(i) $\lambda i(\gamma i_{\langle \pi i\rangle} \ddot{\vee}\ \delta i_{\langle \neg\pi i\rangle})$,
(ii) $\lambda i(A(\gamma i_{\langle \pi i\rangle}) \ddot{\vee}\ \delta i_{\langle \neg\pi i\rangle})$,
(iii) $\lambda i(\gamma i_{\langle \pi i\rangle} \ddot{\vee}\ A(\delta i_{\langle \neg\pi i\rangle}))$,
(iv) $\lambda i(A(\gamma i_{\langle \pi i\rangle}) \ddot{\vee}\ A(\delta i_{\langle \neg\pi i\rangle}))\}$

Here (i) is the top representation of TS(27). It is easily seen that it presupposes a contradiction, and hence (i) does not meet the consistency requirement. What about the two next representations in line: (ii) and (iii)? Some inspection will learn that these too can never be True and hence violate the consistency requirement as well. This leaves us with (iv), in which both presuppositions are wiped-out/meta-asserted; this expression meets both the consistency and the informativity condition, thus it is rightly predicted to be the correct reading of example (27).

Let us now illustrate the informativity condition. Reconsider (8), discussed above as a counterexample to the cumulative hypothesis embodied by the weak Kleene system and repeated below as (28).

(28) If Mary is sad, then Bill regrets that Mary is sad.

Schematically, this sentence would be represented by the following formula: $\lambda i(\pi i \mathrel{\ddot{\to}} \delta i_{\langle \pi i\rangle})$, where π is the proposition that Mary is sad and δ the proposition that Bill regrets this. The translation set contains two expressions:

(i) $\lambda i(\pi i \mathrel{\ddot{\to}} \delta i_{\langle \pi i\rangle})$
(ii) $\lambda i(\pi i \mathrel{\ddot{\to}} A(\delta i_{\langle \pi i\rangle}))$

Some inspection shows that (i) violates the informativity constraint. If we replace the antecedent πi with $\top$ we end up with a formula which is True in precisely the same circumstances as (i) itself. In (ii) the presupposition is wiped out and the two conditions are met. Hence this expression is predicted to be the correct representation of (8), and it does not presuppose that Mary is sad.

[16] C.f. the discussion in Horn (1985:125) of lexical vs. structural ambiguity as explaining projection behaviour of negative sentences.

5.3. Accommodation and Common Ground

Next, we consider another way in which the partial account may be extended, so as to incorporate a notion of accommodation. The accommodation mechanism we describe is independent from the floating-A theory, a separate line of investigation. In fact, we believe it would be natural to consider the floating-A theory as achieving what in Heim and van der Sandt's theory is achieved by so-called *local accommodation*, but the accommodation mechanism we now describe as pertaining to what Heim and van der Sandt call *global accommodation*. Thought of this way, it would be natural to utilise a combination of the floating-A and accommodation mechanisms we propose within a single theory. But to justify such a move would require a more extensive consideration of the data than we make here. For the moment we are content merely to show that there are some unexplored lines of research.

We begin our discussion of accommodation with the following question: how can a partial account of presupposition be used to account for data based on utterance (in-)felicity? Felicity, as regards cases of so-called *presupposition failure*, does not depend on how the world is, but on what we know about it. A sufficient condition for utterance infelicity might be that there is some presupposition which is mutually believed to be false by speaker and hearer.

Suppose that we represent the common ground (mutual beliefs, pragmatic presuppositions) as a set of possible worlds, σ. Then after an utterance of a sentence S which is true in the worlds S_t, we should expect the common ground to be characterized by $\sigma \cap S_t$. But a necessary condition for common ground update to occur (the reverse of the above sufficient condition for infelicity, which here we identify with failure to enable an update) is that the common ground supports all the presuppositions. If the set of worlds where the sentence fails to be true or false is $S_\star$, then we obtain the following definition of update:

$$\sigma + S = \sigma \cap S_t \text{ if } \sigma \cap S_\star = \emptyset$$
$$\text{otherwise } \sigma + S \text{ is not defined}$$

In Beaver (1995) it is argued that what is important for felicity is not the common ground *per se*, but what the speaker takes (or appears to take) the common ground to be. As long as there is some plausible choice of an initial common ground which can be updated with each successively uttered sentence in a given monologue, then the discourse will be felicitous. It follows that we can see a discourse as providing two sorts of information: information as to what the initial common ground was taken to be, and information as to what the final common ground is expected to be. The initial common ground is constrained by presupposition, and the relation between the initial and final common ground is constrained by assertion. As observers, we cannot tell just by looking at a series of sentences what the initial or final common grounds of the interlocutors was or was taken to be, but we can limit the options that are consistent with the discourse successfully having updated that common ground. Suppose that Σ is a set of initially possible common grounds, each of the possibilities itself being classified (*à la Stalnaker*) as a set of worlds. Then after each sentence there will be a new set of possible common grounds which may be calculated through the following two stage procedure: firstly filter out those members of Σ which are incompatible with the presuppositions of the sentence, and then update each of the remaining possible initial common grounds with the assertion. So if the new set of common grounds is denoted $\Sigma + S$, then we obtain:

$$\begin{aligned}\Sigma + S &= \{\sigma \mid \exists\tau \in \Sigma : \tau + S \text{ is defined and equal to } \sigma\}\\ &= \{\sigma \mid \exists\tau \in \Sigma : \tau \cap S_\star = \emptyset \text{ and } \tau \cap S_t = \sigma\}\end{aligned}$$

In a monologue situation, a hearer will be in essentially the same position as any watching linguist, the situation of not knowing what the speaker takes the common ground to be. So the above definition models the information that any hearer will gain as a result of update with a given sentence. The process whereby the hearer gains information via presuppositions is normally referred to as *accommodation*, following Lewis (1979), albeit that this process is commonly conceived of as a sort of erase-and-rewrite operation on information states rather than as a filtering operation.

The above model of accommodation provides a transition from the partial semantics of sentence meaning to the (dynamic) pragmatics of information update. But it also provides a framework in which to account for certain inferences that hearers make. For instance, note that whilst in the case of the Spaceman Spiff example above (25) a conditional presupposition (that if Spiff lands on X his weight will be higher than on Earth), the presupposition apparently associated with the following example is different:

(29) If Spaceman Spiff lands on the weighing scale, he will be bothered by the fact that his weight is higher than it was yesterday.

Here we seem to conclude not that if Spiff lands on the scale his weight will be higher than yesterday, but that his weight is higher *simpliciter*, regardless of whether he lands on the scale or not. This inference could perhaps be explained if it were assumed that the implausibility of Spiff's weight being dependent on that weight being measured results in a limitation on what are considered as *initially plausible common grounds*. In particular, we would have to assume that there are no plausible initial common grounds such that in those worlds where Spiff is weighed he is heavier than those otherwise similar worlds in which he is not weighed.

Now this analysis is of course very tentative. And it is clear that in a full model we would not want to depend on an absolute line drawn between plausible and implausible common grounds, but on some sort of relative grading of the plausibility of different common grounds[17]. But it should be clear that adding a dynamic model of accommodation provides yet another way in which a partial theory of presupposition can be made more flexible.

6. Conclusion

In this paper we have been concerned to show that the most traditional of approaches to presupposition remains feasible and open to further lines of development. In particular, we have shown that natural extensions of partial propositional logics to deal with quantification yield systems having desirable properties from the point of view of the interaction between presuppositions and quantifiers, and we have shown how Montague Grammar may be best adapted to the needs of a partial treatment of presupposition based on the underlying logic of these systems.

Going beyond these concrete results, we have discussed some obstinate points of criticism which have been levelled at accounts of presupposition projection using partial logic. We have shown that these in fact do not provide insurmountable problems for the partial approach. We have explored some possible ways of extending a partiality based treatment of presupposition. Let us finish by summarizing the conclusions we wish to draw from these last explorations. We have argued that it is possible that a partial account of presupposition might be given the sort of flexibility needed to account for a range of counterexamples to traditional partiality-based theories. But the danger of making this move is of creating a theory which is so flexible that it also introduces unwanted readings, and thus new counterexamples. To control this flexibility, a method of constraining readings is necessary. We have discussed two common constraints, namely informativity and consistency. But more constraints are likely to play a role. Similarly, with regard to the proposed model of global accommodation, the exact predictions of the extended model will depend on exactly what is accommodated, and this in turn will depend on the notion of *plausible initial common ground*. But we have not provided any discussion of the issue of what should constitute a plausible initial common ground.

We accept that in a fully developed theory the constraints on readings, and on common grounds, may take up a considerable burden. It is clear that constraints on readings would ultimately have to take into account a range of pragmatic considerations, such as *coherence* of the discourse as a whole.[18] And it

[17] In fact the apparatus needed to make global accommodation dependent on the relative plausibility of different common grounds is developed in Beaver (1997b).

[18] Consider, e.g., the common observation that cancellation examples typically occur within denials, and then with marked intonation. See Blok (1993) and Sandt (ms.) for discussion. To predict when a cancellation reading is available we will have to take into account not only whether other readings are consistent with established knowledge, but also whether the discourse context and intonation contour allows for a cancellation reading.

is clear that any account of the constraints on common grounds would ultimately involve a discussion of the role of world knowledge and default assumptions. It should be stressed however, that in this respect the partial approach described in this paper is not different from other current theories of presupposition. For example, the account of van der Sandt (1992), in many respects the most empirically successful of contemporary presupposition theories (Beaver 1997a), does not involve any formal reference to world knowledge or default knowledge. In fact, there is to the best of our knowledge no theory of presupposition which employs a full-fledged pragmatic component dealing with the influence of coherence, common ground, world knowledge and default assumptions and their relation with presupposition projection.

The question of what role would remain for partial logic, given further sophisticated pragmatic extensions, must for now remain open. We can say that the possibility of pragmatic extensions such as we have described shows that none of the examples standardly conceived as problematic in fact provides a knock-down argument against partiality based accounts of presupposition. And we can say, a century after Frege initiated the partial treatment of presupposition, that there remain some promising and largely unexplored areas for future research.

The partial treatment of presupposition would be worth pursuing if only because it can continue to teach us lessons that inform technically related alternatives, such as those utilising a dynamic semantics. Having considered elsewhere the strengths and weaknesses of a range of contemporary theories of presupposition (Beaver 1997a), we believe a stronger claim is in order. A suitably developed partial treatment of presupposition can match or better the empirical coverage of any alternative yet proposed.

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Appendix

A. The fragment

This appendix lists all the relevant definitions which together form *Presuppositional Montague Grammar*. As in the main text, the emphasis will be on the semantics. For an extensive presentation of *both* the syntax *and* the semantics of classical Montague Grammar in terms of type theory the reader is referred to Muskens (1989). Extensions of the basic fragment Presuppositional Montague Grammar fragment can be found in Krahmer (1998). The set of categories is defined as follows:

DEFINITION 18 (Categories)

1. *E* is a category; *S* is a category;
2. If *A* and *B* are categories, then *A*/*B* and *A*//*B* are categories.

The following table lists the categories that we actually use in the fragment.

Category	*Abbreviation*	*Basic Expressions*
S		
S/*E*	*VP*	whistle, be rewarded
S//*E*	*CN*	man, bicycle, king
S/*VP*	*NP*	Bill, Mary, George V, somebody, t_n
VP/*VP*		try to, manage to
VP/*NP*	*TV*	succeed, push, love, serve
NP/*CN*	*DET*	every, a, the
CN/*CN*	*ADJ*	fat
DET/*NP*	*POSS*	's

We only use three of Montague's syntactic rules (besides the basic rule): functional application, quantifying-in and relative clause formation.

DEFINITION 19 (Syntactic Trees)

1. BASIC:
   If α is a basic expression of category A, then $[\,\alpha\,]^A$ is a tree.
2. FUNCTIONAL APPLICATION:
   If $[\,\alpha\,]^{A/^m B}$ and $[\,\beta\,]^B$ are trees, then $[[\,\alpha\,]^{A/^m B}\ [\,\beta\,]^B]^A_{\mathrm{fa}}$ is a tree ($m \in \{1,2\}$).
3. QUANTIFYING-IN:
   If $[\,\xi\,]^{NP}$ and $[\,\vartheta\,]^S$ are trees, then $[[\,\xi\,]^{NP}\ [\,\vartheta\,]^S]^S_{\mathrm{qi},n}$ is a tree, for $n \in I\!N$.
4. RELATIVE CLAUSE FORMATION:
   If $[\,\xi\,]^{CN}$ and $[\,\vartheta\,]^S$ are trees, then $[[\,\xi\,]^{CN}\ [\,\vartheta\,]^S]^{CN,n}_{\mathrm{rcf}}$ is a tree, for $n \in I\!N$.

Let us now focus on the semantics. The following definition maps (syntactic) categories to (semantic) types.

DEFINITION 20 (Category-to-type Rule)

1. $\mathsf{TYPE}_2(E) = e; \mathsf{TYPE}_2(S) = (st)$;
2. $\mathsf{TYPE}_2(A/B) = \mathsf{TYPE}_2(A//B) = (\mathsf{TYPE}_2(B)\mathsf{TYPE}_2(A))$.

We use the following terms in the representations:

Type	*Constants*	*Variables*
e	*b,m,g*	x, y
s		i, j
st		$\mathcal{P}$
$e(st)$	*whistle, reward*	P_i
$e(st)$	*man, bike, king*	P_i
$e(e(st))$	*succeed, push, love, serve, of*	
$(e(st))(st)$		Q_i
$(e(st))(e(st))$	*try, difficult*	
$(e(st))(e(st))$	*fat*	

Finally, the function $(.)^\bullet$ gives us the translation of the syntactic trees in TY_2^3.

DEFINITION 21 (Translation)
For each tree $[\,\xi\,]$ define its translation $\xi^\bullet$ as follows:

1. BASIC
   whistle$^\bullet$ = *whistle*, be rewarded$^\bullet$ = *reward* ;
   man$^\bullet$ = *man*, bicycle$^\bullet$ = *bike*, king$^\bullet$ = *king*;
   Bill$^\bullet$ = $\lambda P(P\ b)$, George V$^\bullet$ $= \lambda P(P\ g)$, Mary$^\bullet$ $= \lambda P(P\ m)$,
   somebody$^\bullet$ $= \lambda P\lambda i\exists x(P\ xi)$, $t_n{}^\bullet = \lambda P(P\ x_n)$;
   try to$^\bullet$ = *try*; manage to$^\bullet$ $= \lambda P\lambda x\lambda i(P\ xi_{\langle(\mathit{difficult}\ P)\ xi\rangle})$;
   succeed$^\bullet$ $= \lambda Q\lambda y(Q\lambda x(\mathit{succeed}\ xy))$, push$^\bullet$ $= \lambda Q\lambda y(Q\lambda x(\mathit{push}\ xy))$,
   love$^\bullet$ $= \lambda Q\lambda y(Q\lambda x(\mathit{love}\ xy))$, serve$^\bullet$ $= \lambda Q\lambda y(Q\lambda x(\mathit{serve}\ xy))$;
   every$^\bullet$ $= \lambda P_1\lambda P_2\lambda i\forall x(P_1\ xi \dot{\rightarrow} P_2\ xi)$,
   a$^\bullet$ $= \lambda P_1\lambda P_2\lambda i\exists x(P_1\ xi \dot{\wedge} P_2\ xi)$,
   the$^\bullet$ $= \lambda P_1\lambda P_2\lambda i(\exists x(P_1\ xi \dot{\wedge} P_2\ xi)_{\langle\exists!xP_1\ xi\rangle})$;
   's$^\bullet$ $= \lambda Q\lambda P_1\lambda P_2\lambda i$
   $(\exists x(P_1\ xi \dot{\wedge} Q\lambda y(\mathit{of}\ yx)i \dot{\wedge} P_2\ xi)_{\langle\exists!x(P_1\ xi \dot{\wedge} Q\lambda y(\mathit{of}\ yx)i)\rangle})$;
   fat$^\bullet$ $=$ *fat*;
2. FUNCTIONAL APPLICATION
   $([\xi\vartheta]_{\mathrm{fa}})^\bullet = \xi^\bullet\vartheta^\bullet$;
3. QUANTIFYING-IN
   $([\xi\vartheta]_{\mathrm{qi},n})^\bullet = \xi^\bullet\lambda x_n(\vartheta^\bullet)$;
4. RELATIVE CLAUSE FORMATION
   $([\xi\vartheta]^n_{\mathrm{rcf}})^\bullet = \lambda x_n\lambda i(\xi^\bullet\ x_n i \dot{\wedge} \vartheta^\bullet\ i)$.

B. Calculating presuppositions

In this appendix we briefly describe the method to determine maximal presuppositions via the TR^+ and TR^- function. We will illustrate it for the system of *strong Kleene based Partial Predicate Logic* (PPL), which subsumes partial propositional logic and is compatible with TY_2^3. The syntax of PPL is closely related to that of ordinary Predicate Logic (PL): every PL formula is also a PPL formula, but additionally PPL contains a binary presupposition operator. As we have done throughout the paper, we will assume that the only source of partiality is presupposition failure. Thus models M are standard: $M = \langle D, I \rangle$, where D is a non-empty set and I the interpretation function mapping n-place predicates to subsets of D^n. Furthermore, G is the set of total assignments. $g[d/x]$ is the assignment which differs at most from g in that $g[d/x](x) = d$. Terms are interpreted as follows: for $t \in \mathsf{VAR}$, $[\![t]\!]_{M,g} = g(t)$ and for $t \in \mathsf{CON}$, $[\![t]\!]_{M,g} = I(t)$. We define $[\![.]\!]_{M,g}^{PPL} \in \{\mathrm{T, F, N}\}$ as follows, dropping sub- and superscripts where possible.

DEFINITION 22 (Strong Kleene based interpretation of PPL)

1. $[\![R(t_1, \ldots, t_n)]\!]_g = \mathrm{T}$, if $\langle [\![t_1]\!]_g, \ldots, [\![t_n]\!]_g \rangle \in I(R)$
   $[\![R(t_1, \ldots, t_n)]\!]_g = \mathrm{F}$, if $\langle [\![t_1]\!]_g, \ldots, [\![t_n]\!]_g \rangle \notin I(R)$
2. $[\![t_1 \equiv t_2]\!]_g = \mathrm{T}$, if $[\![t_1]\!]_g = [\![t_2]\!]_g$
   $[\![t_1 \equiv t_2]\!]_g = \mathrm{F}$, if $[\![t_1]\!]_g \neq [\![t_2]\!]_g$
3. $[\![\neg\varphi]\!] = -[\![\varphi]\!]$
4. $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$
5. $[\![\forall x\varphi]\!]_g = \bigcap_{d \in D} [\![\varphi]\!]_{g[d/x]}$
6. $[\![\varphi \equiv \psi]\!] = \mathrm{T}$, if $[\![\varphi]\!] = [\![\psi]\!]$
   $[\![\varphi \equiv \psi]\!] = \mathrm{F}$, if $[\![\varphi]\!] \neq [\![\psi]\!]$
7. $[\![\varphi_{\langle\pi\rangle}]\!] = \mathrm{T}$, if $[\![\pi]\!] = \mathrm{T}$ and $[\![\varphi]\!] = \mathrm{T}$
   $[\![\varphi_{\langle\pi\rangle}]\!] = \mathrm{F}$, if $[\![\pi]\!] = \mathrm{T}$ and $[\![\varphi]\!] = \mathrm{F}$

As usual, $\varphi \vee \psi$ is defined as $\neg(\neg\varphi \wedge \neg\psi)$, while $\varphi \rightarrow \psi$ is defined as $\neg(\varphi \wedge \neg\psi)$. $\exists x\varphi$ is defined as $\neg\forall x\neg\varphi$. Two functions are defined, TR^+ and TR^-, where $\mathsf{TR}^+(\varphi)$ produces a PL formula (by definition without elementary presuppositions) which is true iff φ is True, and $\mathsf{TR}^-(\varphi)$ produces a predicate logical formula which is true iff φ is False. $\mathsf{PR}(\varphi)$ is now defined as $\mathsf{TR}^+(\varphi) \vee \mathsf{TR}^-(\varphi)$. These $\mathsf{TR}^\pm$ translations are variants of well-known embeddings of partial logics into standard, total ones as found in e.g., Gilmore (1974), Feferman (1984), Langholm (1988) and others. We let φ_{at} be an atomic formula (either $t_1 \equiv t_n$ or $R(t_1, \ldots, t_n)$). For ease of use we also give the translation functions for disjunction, implication and existential quantification.

DEFINITION 23 (Strong Kleene based TR^+ and TR^-)

$$\begin{array}{lcl}
\mathsf{TR}^+(\varphi_{at}) & = & \varphi_{at} \\
\mathsf{TR}^-(\varphi_{at}) & = & \neg\varphi_{at} \\[1ex]
\mathsf{TR}^+(\neg\varphi) & = & \mathsf{TR}^-(\varphi) \\
\mathsf{TR}^-(\neg\varphi) & = & \mathsf{TR}^+(\varphi) \\[1ex]
\mathsf{TR}^+(\varphi \wedge \psi) & = & \mathsf{TR}^+(\varphi) \wedge \mathsf{TR}^+(\psi) \\
\mathsf{TR}^-(\varphi \wedge \psi) & = & \mathsf{TR}^-(\varphi) \vee \mathsf{TR}^-(\psi) \\[1ex]
\mathsf{TR}^+(\forall x\varphi) & = & \forall x\mathsf{TR}^+(\varphi) \\
\mathsf{TR}^-(\forall x\varphi) & = & \exists x\mathsf{TR}^-(\varphi) \\[1ex]
\mathsf{TR}^+(\varphi \equiv \psi) & = & \mathsf{TR}^+(\varphi) \leftrightarrow \mathsf{TR}^+(\psi) \\
\mathsf{TR}^-(\varphi \equiv \psi) & = & \neg\mathsf{TR}^+(\varphi \equiv \psi) \\[1ex]
\mathsf{TR}^+(\varphi_{\langle\pi\rangle}) & = & \mathsf{TR}^+(\pi) \wedge \mathsf{TR}^+(\varphi) \\
\mathsf{TR}^-(\varphi_{\langle\pi\rangle}) & = & \mathsf{TR}^+(\pi) \wedge \mathsf{TR}^-(\varphi)
\end{array}$$

The following fact can be proven by an easy induction:

FACT 3 (From PPL to PL) For all models M and assignment g, and for all PPL formulae φ:

1. $M \models \mathsf{TR}^+(\varphi)[g]$ iff $[\![\varphi]\!]^{PPL}_{M,g} = \mathrm{T}$
2. $M \models \mathsf{TR}^-(\varphi)[g]$ iff $-[\![\varphi]\!]^{PPL}_{M,g} = \mathrm{T}$

Where $\models$ is the usual Tarskian truth-condition for PL. Since $\mathsf{PR}(\varphi)$ is a formula with elementary presuppositions (and given our assumption that partiality only arises in the case of presupposition failure) it is immediately seen that $\mathsf{PR}(\varphi)$ is a true PL formula iff $\mathsf{PR}(\varphi)$ is a True PPL formula.