ECON 387
Spring 2018

Comprehensive Exam

There are 4 questions worth 45 points each.

1. Overlapping Generations The economy has time running from $t = 0, 1, ...$ At each date there are three generations alive with an equal number of people in each- the young (y), middle aged (m), and old (o). Endowments are stationary over time, but differ by age, with $e_m > e_o \geq e_y$. Preferences are given by:

$$U^t\left(\left(c_\ell\right)_{\ell=0}^{\infty}\right) = \log c_t + \log c_{t+1} + \log c_{t+2}.$$

- Define a sequential markets equilibrium in which households can buy or sell a financial asset with price q_t.
- Characterize all stationary equilibria in this economy. Explain fully whether or not autarky is an equilibrium.

Now assume that there is an aggregate shock that shifts all endowments by the same amount. That is, $e_{i,t}(z^t) = e_i z_t$ with $e_m > e_o \geq e_y$ as above. Denote the probability of history z^t by $\pi(z^t)$ and the conditional probability of event z_{t+1} after history z^t by $\pi\left(z_{t+1}|z^t\right)$.

- Define a sequential markets equilibrium with a full set of state contingent Arrow securities and show that consumption growth is equalized among all generations alive after history z^t:

  $$\forall z^t, z_{t+1} : \frac{c_{t+1}^t\left((z_{t+1}, z^t)\right)}{c_t^t\left(z^t\right)} = \frac{c_{t+1}^{t-1}\left((z_{t+1}, z^t)\right)}{c_t^{t-1}\left(z^t\right)},$$

  where $c_j^t(z^j)$ is the consumption of someone born at t in period $j > t$ after history z^j.

Now suppose that the only asset available is a risk-free bond, so that budget constraints are given by:

$$\begin{aligned} c_t^t(z^t) + Q_t(z^t)A_t^t(z^t) &= e_y z_t \\ c_{t+1}^t(z^{t+1}) + Q_t(z^{t+1})A_{t+1}^t(z^{t+1}) &= e_m z_{t+1} + A_t^t(z^t) \\ c_{t+2}^t(z^{t+2}) &= e_o z_{t+2} + A_{t+1}^t(z^{t+1}). \end{aligned}$$

- Define an equilibrium in this economy and show how the risk-sharing condition fails in this case due to the non-contingent financial securities.
- Now introduce "handsome cat figurines" to this economy. These are little statues that provide no dividend or any other intrinsic value. Show that there is an equilibrium of this economy in which the risk-sharing condition holds when the figurines sell at price $P_t^f(z^t) = Pz_t$ for some $P > 0$ (that is, you buy a figurine in t at price Pz_t and sell it in $t+1$ at price Pz_{t+1}). Assume that you can short-sell figurines.

2. Neo-Classical Growth Model With Heterogeneous Labor This economy has two households who live for $t = 0, 1, ...$, indexed by the productivity of their labor, with $i \in \{L, H\}$. The households each have utility functions of the form:

$$\sum_{t=0}^{\infty} \beta^t \Big[\log c_t + v(1 - h_t) \Big],$$

where h_t is the amount of time spent working in period t. The production technology has constant returns to scale in the two types of labor and capital, with

$$Y_t = F\big(K_t, Z_{Ht} h_{Ht}, Z_{Lt} h_{Lt}\big),$$

where Z_{it} is the productivity of worker type i in period t and capital accumulates as $k_{it+1} = (1 - \delta) k_{it} + x_{it}$ and F is a CRS function.

- Denoting the wage of worker type i by w_{it}, define and characterize an Arrow-Debreu equilibrium in this economy.
- Show that the above equilibrium is Pareto Efficient and solves a Social Planner's Problem for appropriate weights on each worker type.
- Suppose that $Z_{Ht} = 1$ and $Z_{Lt} = \zeta < 1$ for all t. Show that the planner's problem can be written as a stationary dynamic programming problem. Provide conditions on v and F so that the optimal value and policy functions exist and are unique. Use these to characterize the steady state of this economy.
- Now suppose that $Z_{Ht} = (1 + \gamma)^t$ and $Z_{Lt} = \zeta(1 + \gamma)^t$ with $\zeta < 1$. Can you detrend consumption, investment, and capital so that the economy has an equilibrium with a balanced growth path in which all variables grow at rate γ over time (except for hours, which are constant)?

3. Depreciation shock in a RBC model (45 points)

Consider the following closed-economy RBC model. There is no growth for simplicity.

Preferences

The representative household's expected discounted utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right)$$

where E_0 is the conditional expectation operator, C_t is consumption, L_t is leisure, and $0 < \beta < 1$ is the discount factor. The period utility $u\left(\cdot\right)$ is strictly increasing, concave, and twice continuously differentiable.

Production Technology

In this economy, output (Y_t) is produced using a production function

$$Y_t = F(K_t, N_t)$$

where K_t is (pre-determined) capital and N_t is labor. The production function $F(\cdot)$ is twice continuously differentiable, concave, and homogenous of degree one. $F(\cdot)$ also satisfies the standard limiting conditions (the Inada conditions).

Accumulation Technology

The evolution of capital K_t is given by

$$K_{t+1} = I_t + \left(1 - \delta_t\right) K_t$$

where I_t is investment and where the rate of depreciation δ_t follows an exogenous stationary, stochastic process.

Resource Constraints

The total amount of time that the household has can be split into work and leisure. Normalizing the total amount of time each period to be 1, the time constraint is

$$N_t + L_t = 1.$$

Moreover, since total output produced can be either consumed by the household and government or invested, another resource constraint is

$$Y_t = C_t + I_t.$$

(i) Formulate a price-taking version of the above model in which *the representative firm owns the capital stock and issues shares to finance investment.*

(ii) Define the competitive equilibrium based on your formulation in (i) above and derive all the conditions that characterize it.

(iii) Let $\hat{\delta}_t$ generally follow a stationary AR(1) process, where $\hat{\delta}_t$ is the deviation of δ_t from the non-stochastic steady-state. Without necessarily explicitly deriving the solution, describe the economic reasoning behind the response of output, consumption, hours, and investment to a shock to $\hat{\delta}_t$.

In your answer, first describe the results where $\hat{\delta}_t$ is iid and then discuss how results might change when it is very persistent.

(You do not have to linearize all the equilibrium conditions explicitly. You can however, discuss the results in terms of a linearized version of some of the equilibrium conditions, as necessary. You can also assume log-separable preferences and a Cobb-Douglas production function if it is helpful to describe your results and intuition.)

(iv) Formulate the planner's problem for the model given above.

(v) Do the allocations from the planner's problem and the competitive equilibrium coincide? Defend your answer fully.

4. Optimal monetary policy in a sticky price model (45 points)
The central bank's objective is to minimize the loss function

$$\frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^j\left[\phi_\pi\pi_{t+j}^2+\phi_x x_{t+j}^2+\phi_i i_{t+j}^2\right]$$

subject to

$$\pi_t=(1-\lambda)\,\beta E_t\pi_{t+1}+\kappa x_t+\lambda\pi_{t-1}+\varepsilon_{\pi,t}$$
$$x_t=E_t x_{t+1}-(i_t-E_t\pi_{t+1})+\varepsilon_{x,t}$$

where E_t is the conditional expectation operator, i_t is the central bank's instrument, π_t and x_t are other endogenous model variables, and $0<\beta<1$, $0<\lambda<1$, $\kappa>0$, $\phi_\pi>0, \phi_x>0, \phi_i>0$ are model parameters. The central bank takes actions after the shocks $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$ are realized. The shocks $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$ are iid over time and have unit variance.

(i) First, suppose that the central bank can credibly commit at date t to a contingent path for i_{t+j}. Characterize, as far as you can, the solution to the optimal monetary policy problem above with commitment. Does the solution feature dynamic time-inconsistency? Defend your answer.

(ii) Next, suppose that the central bank cannot credibly commit and, instead, chooses i_t at each date. Characterize, as far as you can, the (Markov-perfect) solution to the optimal monetary policy problem above without commitment.

(iii) Finally, consider a simplified case where $\lambda=0$ and where the central bank can credibly commit at date t to a contingent path for i_{t+j}. Without necessarily explicitly deriving the solution, describe the economic reasoning behind the response of π_t, x_t, and i_t to shocks $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$.

Additionally, describe how your answer would simplify if also $\phi_i=0$.