The University of Texas at Austin
Department of Economics

Microeconomics
Comprehensive Examination
June 2021

Instructions:

1. Please answer each of the four questions on separate pieces of paper.
2. Please write only on one side of a sheet of paper.
3. Please write in pen only.
4. When finished, please arrange your answers in the order in which they appeared in the questions, i.e. 1(a), 1(b), etc.

Question 1: Consider a pure exchange economy with two consumers A and B and two goods 1 and 2 and utility functions:

$$u_A = 2\sqrt{x_1^A} + 2x_2^A, u_B = 3x_1^B + \sqrt{x_2^B},$$

where x_i^j denotes the consumption of good $i \in \{1, 2\}$ by consumer $j \in \{A, B\}$. Both of them consume positive amounts and are price takers. Suppose $e^A = (2, 0)$ and $e^B = (0, 2)$.

(a) Carefully write the definition of competitive equilibrium for this economy.

(b) For any market price ratio $p = \frac{p_1}{p_2}$, solve the utility maximization problem of each consumer and find Walrasian demands. Be careful to argue which of the consumer's constraints bind and why.

(c) Find all (if any) equilibrium price ratios and allocations.

(d) Write down an appropriate programming problem that characterizes all Pareto optimal allocations in this economy. Be careful to argue which of the constraints bind and why.

(e) Find all Pareto optimal allocations in this economy.

Question 2:

(a) Define the Archimedean axiom for a binary relation on a convex set. Define the axiom of continuity for a binary relation on a convex set. (Hint: Recall a preference relation on a some set Π is a binary relation.)

(b) Prove that if a binary relation is continuous then it is Archimedean.

(c) Find an example of a binary relation that is Archimedean but not continuous.

(d) Carefully define Anscombe-Aumann acts.

(e) Define state-independent preference relations on the space of acts.

(f) Prove that if a preference relation ($\succeq$) on Anscombe-Aumann acts is state-independent, then $\succeq$ is monotonic where monotonicity is defined as follows:

The binary relation $\succeq$ on $(\Delta X)^{\Omega}$ is monotonic if for all $h, g \in (\Delta X)^{\Omega}$,

$$h_s \succeq g_s \ \ \forall s \in \Omega \Rightarrow h \succeq g$$

where h_s: denotes the <u>constant</u> function that assigns the lottery $h_s \in \Delta X$ to all states $s \in \Omega$. (In words: an act that is preferred in each state must be preferred overall–it is like state by state dominance).

Hint: assume 2 states first.

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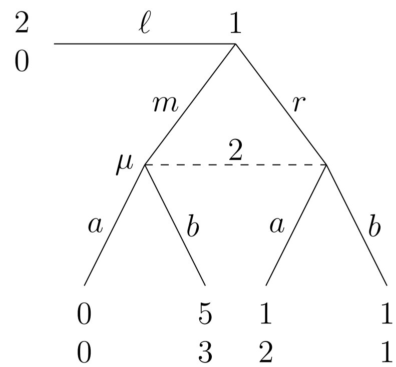
Question 3:

(a) In the extensive-form game below,

i. find all pure strategy Nash equilibria,

ii. find all pure strategy subgame perfect equilibria,

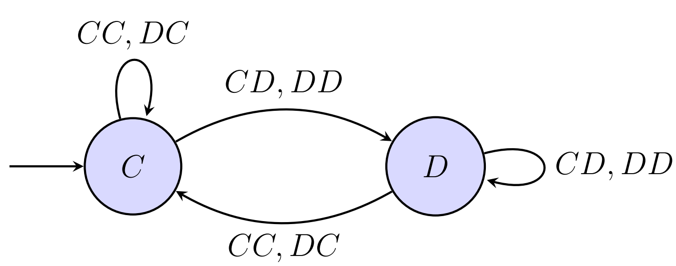
iii. define consistent beliefs in the context of this game, then find all pure strategy sequential equilibria.



(b) Consider the prisoners' dilemma with payoffs given in the matrix below. Suppose it is repeated infinitely often. A player's payoff in the infinitely repeated game is the average discounted sum of stage-game payoffs, $(1-\delta)\sum_{t=0}^{\infty}\delta^t\, u(\mathbf{a}^t)$, where $\delta \in (0,1)$ is the common discount factor.

	C	D
C	5,5	$0,6$
D	$6,0$	1,1

We call "tit-for-tat" the strategy described in the automaton below. For what discount factors, if any, is the strategy profile (tit-for-tat, tit-for-tat) a subgame perfect equilibrium of the infinitely repeated game? (Hint: Divide the set H of possible histories into subsets, according to the action profile that was played in the previous period. Do not forget the empty history.)



Question 4: A single monopolistic employer makes a "take-it-or-leave-it" offer to employ a potential employee. If the employee accepts, he chooses an effort level $e \in \{e^1, e^2\}$. His work produces a profit of $x \in \{x_1, x_2\}$. The probability of the profit realizations depends on the effort according to the following table:

	e^1	e^2
x_1	2/3	1/3
x_2	1/3	2/3

Let p_i^k denote the probability of achieving profit x_i under effort level e^k. Profits are verifiable, but the employer cannot observe the employee's effort. The utility of the employer when she achieves profit x and pays a wage w to the employee is $x - w$. The utility of the employee when he chooses effort e and receives a wage w is $U(w) - v(e)$, where $U(w) = \sqrt{w}$. Assume that $v(e^1) = 1$, $v(e^2) = 2$, $x_1 = 3$ and $x_2 = 24$. The employee's utility in case he rejects the contract offer is $\bar{u} = 0$.

(a) What type of asymmetric information is present in this model? Who is the principal and who is the agent? Determine the risk-attitudes of both parties.

(b) Derive the optimal contract if effort is observable.

(c) Now assume that effort is not observable.

  i. Write down the optimization problem that yields the optimal contract. Describe the role of the constraints.

  ii. Show that the full information contract you derived in (b) cannot be implemented if effort is not observable, because it does not satisfy one of the constraints. Explain in words why this is the case.

  iii. Derive the optimal contract under unobservable effort and show that it is not efficient. Explain in words why the optimal contract involves an inefficiency.