Macroeconomic Theory
Spring 2016

Comprehensive Exam

There are 4 questions and a total of 180 points.

1. Government spending in a RBC model (35 points)

Consider the following closed-economy RBC model. There is no growth for simplicity.

Preferences

The representative household's expected discounted utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right)$$

where E_0 is the conditional expectation operator, C_t is consumption, L_t is leisure, and $0 < \beta < 1$ is the discount factor. The period utility $u\left(\cdot\right)$ is strictly increasing, concave, and twice continuously differentiable.

Production Technology

In this economy, output (Y_t) is produced using a production function

$$Y_t = A_t F(K_t, N_t)$$

where K_t is (pre-determined) capital, N_t is labor, and A_t is a random productivity shock. The production function $F(\cdot)$ is twice continuously differentiable, concave, and homogenous of degree one. $F(\cdot)$ also satisfies the standard limiting conditions (the Inada conditions).

Accumulation Technology

The evolution of capital is given by

$$K_{t+1} = I_t + \left(1 - \delta\right) K_t$$

where I_t is investment and $0 < \delta < 1$ is the rate of depreciation.

Government

The government consumes an *exogenous and random amount* G_t every period. The government finances its consumption using *lump-sum* taxes.

Resource Constraints

The total amount of time that the household has can be split into work and leisure. Normalizing the total amount of time each period to be 1, the time constraint is

$$N_t + L_t = 1.$$

Moreover, since total output produced can be either consumed by the household and government or invested, another resource constraint is

$$Y_t = C_t + I_t + G_t.$$

(i) (10 points) Formulate a price-taking version of the above model in which *the representative firm owns the capital stock and issues equity to finance investment.*

(ii) (5 points) Define the competitive equilibrium based on your formulation in (i) above.

(iii) (5 points) Derive all the conditions (optimality and market clearing) that characterize the competitive equilibrium.

Now suppose that government spending enters household utility. The household's period utility $u(\cdot)$ is then given by

$$u(C_t, L_t, G_t). \tag{1}$$

(iv) (5 points) Depending on the properties of the period utility given in (1), argue carefully if the competitive equilibrium (without necessarily solving for it fully) will lead to different allocations than those above in (iii).

(v) (10 points) Given the period utility in (1), consider now the government *optimally* determining the level of government spending. What principle/optimality condition will determine the level of government spending in this case?

(You do not have to fully solve the model for equilibrium under optimal government spending policy. Note also that the rest of the model, including production and technology technologies and resource constraints, remain the same as in (i)-(iii)).)

2. Optimal monetary policy in a sticky price model (55 points)

The central bank's objective is to minimize the loss function

$$\frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^j\left[\phi_\pi\pi_{t+j}^2+\phi_x x_{t+j}^2+\phi_i i_{t+j}^2\right]$$

subject to

$$(\pi_t-\alpha\pi_{t-1})=\beta E_t\left[\pi_{t+1}-\alpha\pi_t\right]+\kappa\left(x_t-\lambda x_{t-1}\right)+\varepsilon_t$$

$$(x_t-\lambda x_{t-1})=E_t\left[x_{t+1}-\lambda x_t\right]-(i_t-E_t\pi_{t+1})$$

where E_t is the conditional expectation operator, i_t is the central bank's instrument, π_t and x_t are other endogenous model variables, and $0<\beta<1, 0<\alpha<1$, $0<\lambda<1$, $\kappa>0$, $\phi_\pi>0, \phi_x>0, \phi_i>0$ are model parameters. The central bank takes actions after the shock ε_t is realized. ε_t is iid over time and has unit variance.

(i) (20 points) First, suppose that the central bank can credibly commit at date t to a contingent path for i_{t+j}. Characterize, as far as you can, the solution to the optimal monetary policy problem above with commitment. Does the solution feature dynamic time-inconsistency? Defend your answer.

(ii) (35 points) Next, suppose that the central bank cannot credibly commit and, instead, chooses i_t at each date. Characterize, as far as you can, the (Markov-perfect) solution to the optimal monetary policy problem above without commitment.

3. Over-Lapping Generations (45 pts) Consider a Lucas Tree economy with three over lapping generations. That is, for any period $t \geq 0$ there are three households alive, the young, middle-aged, and old. Preferences are given as follows:

$$U^t(c_t^t, c_{t+1}^t, c_{t+2}^t) = \sum_{\ell=0}^{2} \psi_\ell \frac{(c_{t+\ell}^t)^{1-\sigma}}{1-\sigma}$$

Where $\psi_0 = 1$ and ψ_1, ψ_2 are general utility weights.

Households receive endowments only when they are alive, the profile of which is constant: $e^t = (e_y, e_m, e_o)$ with $e_y + e_m + e_o = 1 - \theta$. In addition, there is a Lucas Tree in the economy, which produces θ units of consumption good in each period.
There is an initial old household in $t = 0$ who is endowed with s_o shares of the tree (in addition to e_o units of good) and an initial middle-aged who is endowed with $s_m = 1 - s_o$ units of the tree (in addition to e_m units of good). Young households are always born with zero shares of the tree.

1. Define a Sequential Markets Equilibrium for this economy.

2. Show that the equilibrium of this economy can be characterized by a system of difference equations in $s_{m,t}$ and p_t (the price of the tree).

3. Is it always Pareto Optimal to have consumption constant across age groups? If not, then give a condition for it to be so.

4. Maintain the condition from (3). Can you necessarily find a sequence of prices so that the constant consumption allocation is consistent with equilibrium? If not then give a condition so that the constant consumption allocation is an equilibrium.

4. Optimal Fiscal Policy (45 pts) Consider an economy with a pollution externality. That is, for every unit of output there is some amount of pollution generated, call it $\Phi_t = \phi Y_t$. The production technology is neo-classical with capital and labor as inputs and a constant Solow Residual. Capital is accumulated according to the standard law of motion, and households have life-time utility given by:

$$U = \sum_{t=0}^{\infty} \beta^t \left[u(c_t, \ell_t) - \Phi_t \right]$$

Where $\ell_t \in [0, 1]$ is leisure. The government must finance an exogenous sequence of government expenditures, $(g_t)_{t=0}^{\infty}$.

1. Characterize the set of Pareto Efficient allocations in this economy.
2. Define and characterize the Tax-Distorted Competitive Equilibrium in this economy when proportional taxes are levied on labor and capital income. Households should take all prices, taxes, and aggregate variables as exogenous.
3. Derive the Implementability Condition for this economy and set up the Ramsey Planner's problem.
4. What happens to the capital income tax rate as $t \to \infty$?
5. Suppose that the Ramsey Planner had access to lump-sum taxes. Would the Ramsey Planner use only lump sum taxes? Why or why not.