There are 180 total points available. Note the points allocated to each question.

1. Job Search (45 Pts) Consider a worker who is looking for a job. At the beginning of each period while unemployed, the worker receives a wage offer drawn independently over time from a distribution with cdf F over support $[0, w_{max}]$. If the worker accepts a job then she is employed there for the rest of time. If not then she receives an unemployment benefit and searches again the following period. There is no savings or borrowing, so that $c_t = b$ when unemployed and $c_t = w$ when employed with earnings w. You will study how the design of the unemployment benefit system affects household's problem. Households value consumption linearly within each period and discount consumption t periods in the period by factor β^t with $\beta \in (0, 1)$.

1. Write the Bellman operator for the unemployed and employed. Solve for the employed value in closed form.
2. Prove that there is a unique fixed point of the above operator and that it is increasing in w.
3. Show that the optimal policy of an unemployed worker takes the form of a reservation wage, so that she accepts a job with wage $w \geq w^R$ and rejects any with wage below.
4. Now suppose that the minimum wage eligibility expires after $T + 1$ periods. That is, an unemployed worker gets the benefit b for $t = 0, 1, ...T$ but then zero for $t > T$. Write the Bellman Equation(s) for this problem. What happens to the reservation wage as the worker gets closer to exhausting eligibility (i.e., as $t \to T$)?

2. TDCE in Endogenous Growth Model (45pts) Consider an economy with two forms of capital, physical and human. There is a large number of ex-ante identical households with CRRA utility and common discount factor β. These households accumulate physical capital, k, and human capital, h. Firms use these two types of capital in production using a CRS production function of the form:

$$Y_t = F(K_t, H_t)$$

The firm pays w_t units of final consumption good for each unit of human capital employed and r_t for each unit of physical capital. Each form of capital accumulates by investment using the final good (both are putty-putty). The law of motion of each (at the household level) is given by:

$$k_{t+1} = (1 - \delta_k)k_t + x_t$$
$$h_{t+1} = (1 - \delta_h)h_t + e_t$$

The government must fund a fixed sequence of spending $(g_t)_{t=0}^{\infty}$ using only proportional taxes on each factor, denoted by $(\tau_{k,t}, \tau_{h,t})_{t=0}^{\infty}$.

1. Define a TDCE and characterize it via necessary and sufficient conditions on allocations and prices.
2. Give a condition for there to be positive long-run growth in this economy when $\tau_{k,t} = \tau_k$ and $\tau_{h,t} = \tau_h$.
3. Derive the implementability condition(s) for the TDCE of this economy.
4. How would the Ramsey planner set $\tau_{k,t}$ in relation to $\tau_{h,t}$? What happens as $t \to \infty$?

3. Optimal price-setting under costly adjustment (35 points)

Consider a model with monopolistic competition where a continuum of firms (in a unit interval) set nominal prices for differentiated goods while facing price-adjustment costs.

Firm i faces a downward sloping demand curve for its good i given by

$$y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} Y_t$$

where $y_t(i)$ is the output of good i, $p_t(i)$ is the price of good i, and Y_t and P_t are aggregate output and price. The elasticity of substitution across differentiated goods is given by $\varepsilon > 1$. Moreover, aggregate price P_t and output Y_t are given by

$$P_t = \left[\int_0^1 p_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}, \ Y_t = \int_0^1 \left[y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Firm i produces good i with a production technology that uses only labor as input

$$y_t(i) = A_t l_t(i)^{1-\alpha}$$

where $\alpha > 0$, $l_t(i)$ is labor input used by firm i, and A_t is an *iid*, mean-zero, unit-variance aggregate technology shock.

Assume that labor markets in the economy are perfectly competitive and all firms hire labor from the same market.

Changing prices is costly and all firms face a price-adjustment cost $k(.)$ given by

$$k\left(\frac{p_t(i)}{p_{t-1}(i)}\right)$$

where $k(.)$ is convex and $k\,(1) = k'\,(1) = 0$.

(a) Formulate the profit maximization problem of the firm(s).

(b) Characterize the (first-order) optimality condition of the maximization problem above.

(c) Focus on a symmetric equilibrium where all firms choose the same optimal price, produce the same output, and hire the same amount of labor. Under this assumption, derive an aggregate non-linear equation that determines (aggregate) inflation dynamics in the model.

(d) Log-linearize the aggregate non-linear equilibrium condition from (c) around a zero-(net) inflation, flexible-price, non-stochastic, steady-state. Derive a log-linear equation that determines inflation dynamics.

(Hint: Assume that the firms are owned by a representative household. You do not have to fully consider the representative household's maximization problem; however, feel free to make any additional assumptions on the household side that you feel are necessary.)

4. Optimal monetary policy with and without commitment (55 points)

Consider optimal monetary policy in a closed-economy model with sticky price and wages. The central bank's objective is to minimize the loss function

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{\phi_p}{2} \left(\pi_{t+j}^p\right)^2 + \frac{\phi_w}{2} \left(\pi_{t+j}^w\right)^2 + \frac{1}{2} \left(x_{t+j}\right)^2 \right]$$

subject to

$$\pi_t^p = \beta E_t \left[\pi_{t+1}^p\right] + \kappa_p x_t + \frac{\kappa_p}{2} w_t,$$

$$\pi_t^w = \beta E_t \left[\pi_{t+1}^w\right] + \kappa_w x_t - \frac{\kappa_w}{2} w_t,$$

$$w_t = w_{t-1} + \pi_t^P - \pi_t^w - \eta_t$$

where E_t is the mathematical expectation operator conditional on period-t information (including the shock η_t). x_t is the central bank's instrument which is chosen after η_t is realized at the beginning of the period. The shock η_t follows an AR(1) process given by

$$\eta_t = \rho \eta_{t-1} + \upsilon_t$$

where $\rho \in (0, 1)$ and υ_t is an *iid*, mean-zero, unit-variance innovation.

π_t^p, π_t^w, and w_t are the other endogenous variables in the model. Finally, β, ϕ_p, ϕ_w, κ_p, and κ_w are model parameters.

First, suppose that the central bank can commit at date 0 to a contingent path for x_t.

(a) Characterize, as far as you can, the solution to the optimal monetary policy problem with commitment above.

(b) Argue carefully whether the solution in a) features dynamic time-inconsistency.

Next, suppose that the central bank cannot commit and, instead, chooses x_t at each date.

(c) Characterize, as far as you can, the solution to the optimal monetary policy problem without commitment above.

Finally, consider a special case of the model with $\kappa_p = \kappa_w = \kappa$ and $\phi_p = \phi_w = \phi$.

(d) Under this assumption, characterize fully the solution under optimal policy. Argue carefully whether there is a difference in the solution depending on if the central bank can or cannot commit to a contingent path for future policy.