# Macro I: Comp Exam July 29th, 2021

## 1 Consumption and Asset Pricing (45 points)

Consider an individual seeking to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \tag{1}$$

where $c_t$ denotes consumption and $0 < \beta < 1$ her discount factor and $\sigma > 0$. In every period the individual receives an exogenous income $Y_t$ which evolves according to a two-state Markov chain where $p_{HH}$ is the probability of staying in the high state $Y_H$, and $p_{LL}$ the probability of staying in the low state $Y_L$, where $Y_H > Y_L > 0$. The individual saves/borrows by buying and selling riskless one-period bonds: $B_t$. We assume for now that the gross interest rate on the bond is constant and denoted as $R_t = R$. The individual has to pay a tax $0 < \tau < 1$ on financial wealth at the *beginning* of the period ($R > \tau$). We assume $B_t$ remains bounded so that $\bar{B} \geq B_t \geq \tilde{B}$ in every period.

(a) write down the individual flow budget constraint

(b) write the Bellman equation for this problem: what are the relevant state variables? [Write the expression for the expected value function explicitly using the two state Markov process.]

(c) Show that Bellman operator (T) is a contraction mapping.

(d) Briefly discuss the additional conditions needed for the value function associated to the optimal solution to be unique.

(e) Assume $v$ is differentiable in $B$. Use first order and envelope conditions to derive the Euler equation. Express the Euler equation in time series form. [Ignore the boundaries on $B_t$. Assume an interior solution.]

(f) So far we considered an individual facing a constant interest rate. Now consider a large number of the same individuals with *idiosyncratic* income processes as described above interacting in perfectly competitive markets. Bonds are in zero net supply. What is the behavior of equilibrium aggregate consumption in this constant interest rate economy? Discuss. [No need for lengthly calculations, if at all.]

(g) Conversely, assume now a representative agent economy with the same income process. What is the behavior of the equilibrium interest rate in this economy?

(h) Consider and economy with a balanced growth path growing at $(1+\gamma), \gamma > 0$ every period (gross growth rate). what is the steady state growth in this economy?

## 2 Question 2: Job search (45 points)

Consider an unemployed worker maximizing

$$E\left\{\sum_{t=0}^{\infty}\beta^t c_t\right\}, \quad 0<\beta<1$$

In each period receives a job offer $w$ and can accept the job or keep searching. Once the job is accepted, the worker keeps working forever. The economy is either in boom (B) or recession (R) and the state of the economy is i.i.d. over time. In particular, in every period there is a probability $p_B$ that the economy is in a boom and a probability $1-p_B$ that it is in recession. This affects the wage distribution faced by the worker. The wage offer is drawn from the IID distribution $F(\nu) = Prob(w \leq \nu), \quad w \in [0, \bar{W}]$. In boom the worker draws from the distribution $F^B$ and in recession $F^R$ where $F^R \leq F^B$. While unemployed the worker receives can engage in more leisure and gets an utility flow of $\psi$; furthermore they can keep searching for a job.

(a) Write the Bellman equation for the unemployment worker.

(b) Define the reservation wage for this problem (without calculating it).

(c) Plot and describe the value function as a function of the reservation wage. Describe the policy function.

(d) Assume the probability of being in boom declines: how does it affect the probability of accepting a job?

(e) What can the government do to return keep the acceptance rate unchanged?

(f) Does the current state of the economy affect the *observed* acceptance rate of a job offer? Does it affect the choice of the worker? Explain.

(g) Suppose the worker is allowed to quit their job in this economy: would they ever do so? Explain.

(h) Suppose now the business cycle changes deterministically every period between booms and recessions. How does the worker's problem change? [no need for a full solution: just discuss how you would approach the problem and speculate about the possible differences relative to the above economy with i.i.d. business cycle.]

## 3 Real business cycle model (45 points)

Consider a price-taking version of a closed-economy business cycle model (with no growth).

The representative household's expected discounted utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \frac{N_t^{1+\phi}}{1+\phi} \right],$$

where $C_t$ is consumption and $N_t$ is hours. Moreover, $0 < \beta < 1$ is the discount factor and $\phi > 0$ is the inverse of the Frisch elasticity of labor supply.

The household owns the capital stock and makes investment decisions. The evolution of capital $K_t$ is given by

$$K_{t+1} = I_t + (1 - \delta) K_t,$$

where $I_t$ is investment, $0 < \delta < 1$ is the rate of depreciation, and $K_0 > 0$ is given.

The firm rents capital and hires labor in perfectly competitive spot markets. The production function of the firm is given by

$$Y_t = X_t K_t^{\alpha} N_t^{1-\alpha},$$

where

$$X_t = A_t K_t^{\gamma}$$

is taken *as given* by the firm and $0 < \alpha < 1$, $\gamma \geq 0$ are parameters. $A_t$ is the standard productivity shock that follows a stationary $AR(1)$ process given by

$$\log A_t = \rho \log A_{t-1} + \varepsilon_{A,t},$$

where $0 \leq \rho < 1$ and $\varepsilon_{A,t}$ is an *iid*, mean zero innovation.

(If it is helpful, you can think here of a large number of identical firms, measure 1, who take $X_t$ as given when solving their maximization problem.)

The resource constraint is given by

$$Y_t = C_t + I_t.$$

(i) Formulate and define carefully the competitive equilibrium of the model described above.

(ii) Characterize all the equilibrium conditions given your formulation above in (i).

(iii) Using economic reasoning, compare the responses (both the on-impact and dynamic) of output, hours, investment, and consumption to the innovation $\varepsilon_{A,t}$ for two different values of $\gamma : \gamma = 0$ and $\gamma > 0$.

(You can consider the log-linearized version of the model in making your argument. Note however, that you do not have to log-linearize or solve the model explicitly in your answer.)

## 4 Optimal monetary policy without commitment (45 points)

The central bank's objective is to minimize the loss function

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\pi_{t+j}^2 + \lambda x_{t+j}^2 + \gamma i_{t+j}^2\right]$$

subject to the constraints

$$\begin{aligned} \pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t, \\ x_t &= \alpha \mathbb{E}_t x_{t+1} + \phi x_{t-1} + \chi x_{t-2} - (i_t - \mathbb{E}_t \pi_{t+1}) + \varepsilon_t, \end{aligned}$$

where $\mathbb{E}_t$ is the conditional expectation operator, $i_t$ is the central bank's instrument, and $0 < \beta < 1$ and $\lambda, \gamma, \alpha, \phi, \chi, \kappa > 0$ are model parameters. Finally, $\pi_t$ and $x_t$ are the other endogenous model variables and $\varepsilon_t$ is an $AR(1)$ shock that follows

$$\varepsilon_t = \rho \varepsilon_{t-1} + \upsilon_t,$$

where $0 < \rho < 1$ and $\upsilon_t$ is an *iid*, mean zero innovation. The central bank takes actions after the innovation $\upsilon_t$ is realized.

Suppose that the central bank *cannot* commit at date 0 to a contingent path for $i_t$ and therefore chooses $i_t$ at each date. Characterize the (Markov-perfect) equilibrium, as far as you can, under this case of no commitment.