

## MICROECONOMICS COMPREHENSIVE EXAM

AUGUST 2017

### INSTRUCTIONS:

- (1) Please answer each of the four questions on **separate** pieces of paper.
- (2) Please write only on **one side** of a sheet of paper
- (3) Please write in **pen only**
- (4) When finished, please arrange your answers **alphabetically** (in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.)

1. Consider an exchange economy with two consumers and two goods. Let  $x_i^j$  denote quantity of good  $j \in \{1, 2\}$  consumed by consumer  $i \in \{1, 2\}$ . Consumers' preferences over consumption goods are given by the following utility functions:

$$\begin{aligned}u_1(x_1^1, x_1^2) &= \alpha \ln x_1^1 + (1 - \alpha)x_1^2, \\u_2(x_2^1, x_2^2) &= (1 - \alpha)x_2^1 + \alpha \ln x_2^2,\end{aligned}$$

where  $\alpha \in (0, 1)$ .

Assume that consumer 1 is endowed with  $\beta$  units of good 1, and consumer 2 is endowed with  $\beta$  units of good 2, where  $\beta > 0$ . The endowment of consumer  $i \in \{1, 2\}$  in good  $\{1, 2\} \ni j \neq i$  is zero. Let  $(p_1, p_2) \gg 0$  denote a vector of market prices.

- (a) Carefully write down the definition of a competitive equilibrium for this economy.
- (b) For any market price ratio  $p = p_1/p_2$ , solve the utility maximization problem of each consumer and find Walrasian demands. Be careful to argue which of the consumer's constraints bind and why.
- (c) Specify the necessary conditions for existence of a Walrasian equilibrium in terms of parameters  $\alpha$  and  $\beta$ . Find equilibria when they exist.
- (d) Will the 1st Basic Welfare Theorem always hold in this economy? Explain why or why not.

2. Consider the same economy as in Question 1, and set  $\alpha = \frac{2}{3}, \beta = 1$ .
- (a) Write down an appropriate programming problem that characterizes all Pareto optimal allocations in this economy. Be careful to argue which of the constraints always bind, never bind, or sometimes bind and why.
  - (b) Find all Pareto optimal allocations in this economy.
  - (c) Will the 2nd Basic Welfare Theorem always hold in this economy? Explain why or why not.

**3.** Consider a firm seeking to hire a new employee. Candidates arrive sequentially, one in each period  $t \in \{1, 2, \dots, T\}$ , and are therefore indexed by  $t$ . The expertise of the candidate arriving at date  $t$ ,  $u_t$ , is an i.i.d. random variable distributed uniformly over the support  $[0, 1]$ . At each  $t$ , the firm can interview candidate  $t$  at cost  $c \in (0, 1/2)$ , or stop its search. If the firm interviews, it observes the realisation of  $u_t$ . If the firm stops its search after having interviewed  $t$  candidates, it can either opt for not hiring any new employee, in which case its payoff is  $z - ct$ , where  $z \in [0, 1 - \sqrt{2c}]$  is the value of the firm's outside option. Or it can hire any of the candidates it has previously interviewed (if there are any). That is, we allow for *recall* in the search process. However, a candidate cannot be hired without being interviewed first. The firm's payoff from hiring candidate  $s \in \{1, \dots, t\}$  after having interviewed  $t$  candidates is  $u_s - ct$ . (Ties: Assume throughout that if the firm is indifferent between its best candidate and the outside option, it opts for its best candidate. If it is indifferent between interviewing a new candidate and stopping its search, it opts for interviewing a new candidate.)

- (1) First suppose that  $T = 1$ . Show that the firm's payoff from interviewing the candidate is  $\frac{1+z^2}{2} - c$ . For what values of  $z$  does the firm wish to interview the candidate at  $t = 1$ ?
- (2) Now suppose that  $T = 2$ ,  $z = 0$  and  $c = 1/8$ . What is the space of pure strategies for the firm? What is the payoff-maximising strategy for the firm? What is the firm's resulting payoff? Does it benefit from being able to interview a second candidate?

4. (1) Let  $k = 1, \dots, K$  be the possible collective decisions, and  $i = 1, \dots, N$  be the set of agents. Let  $v_i(k, \theta_i)$  denote the utility derived by type  $\theta_i \in \Theta_i$  of agent  $i$  when decision  $k$  is chosen. Her type  $\theta_i$  is assumed to be known to  $i$  but not to other agents.

Define the VCG mechanism in this context. Explain why agents have no incentive to misreport their preferences in such a mechanism.

- (2) We specialise the model as follows. There are three agents:  $i \in \{1, 2, 3\}$ , and two indivisible objects,  $A$  and  $B$  to be allocated among these three agents. (Assume that each object must be allocated to an agent) The utilities derived by agent  $i$  if she receives  $A$ ,  $B$  or the bundle  $AB$  are  $v(A, i)$ ,  $v(B, i)$  and  $v(AB, i)$  respectively. If an agent receives no object her utility is zero. The utilities are given in the following table:

	A	B	AB
1	11	2	10
2	4	14	12
3	4	10	9

Define the VCG mechanism for this problem.

- (3) We change the model as follows. Agent 1 values only the bundle  $AB$  according to  $x > 0$ . (Her value for  $A$  or  $B$  separately is 0). Agent 2 values  $A$  but not  $B$ . Her value for  $A$  or  $AB$  is  $y > 0$ . (Her value for  $B$  is 0). Agent 3 values  $B$  but not  $A$ . Her value for  $B$  or  $AB$  is  $z > 0$ . (Her value for  $A$  is 0).

Define the VCG mechanism, depending on whether  $x > y + z$  or  $x < y + z$ . (You may need to distinguish further cases, according to how  $x$ ,  $y$  and  $z$  are ranked.)

- (4) Define the VCG mechanism in the case where  $x = y = z$ . (Observe that the payments of 2 and 3 are 0). Infer a collusive (mutually beneficial) strategy for 2 and 3 when  $x = y + z$ . Show that when  $x = y + z$ , (a) there exists a BNE in which 2 and 3 employ this collusive strategy and 1 reports truthfully, and (b) players 2 and 3 are better off in this BNE than in the truth-telling BNE.

- (5) We change the model as follows. All three agents only value the bundle  $AB$ : agent 1 according to  $x > 0$ , agent 2 according to  $y \in (x, 2x)$  and agent 3 according to  $z \in (x, 2x)$ . (For all three the value of  $A$  or  $B$  separately is 0.) Suppose also that in addition to 1's report about her preferences, she can send two shill bidders (that is, agent 1 can create two fake identities under which she can submit two additional bids) called  $1_A$  and  $1_B$ , where  $1_Z$  pretends to value object  $Z$  according to  $2x$ , the other object at 0, and the bundle also at 0.

Is it advantageous for player 1 to use the shill bidders under the VCG mechanism?