The University of Texas at Austin
Department of Economics

MICROECONOMICS

Comprehensive Examination

June 2019

INSTRUCTIONS:

1. Please answer each of the four questions on separate pieces of paper.
2. Please write only on one side of a sheet of paper.
3. Please write in pen only.
4. When finished, please arrange your answers in the order in which they appeared in the questions, i.e. 1(a), 1(b), etc.

Question 1:

(a) Suppose that Ann has a virtual assistant that determines that the function $F(p_1, p_2, u) = (3p_1 + p_2)u$ describes Ann's optimal spending to achieve utility level u from consuming books (good 1) and music (good 2) with prices p_1 and p_2 respectively. Construct a utility function that generates F.

(b) Suppose that the commodity space is $X \equiv \mathbb{R}^l_+$. Let C be a compact subset of X and let $\succeq$ be a continuous, transitive complete preference relation on X. Provide detailed proofs of the following:

i. There exists a best element $\hat{x} \in C$–that is an element that satisfies $\hat{x} \succeq x$ for all $x \in C$.

ii. The set of best elements given $\succeq$–call it C^*–is compact.

Hint 1: Use the following property of closed subsets of compact sets: If every finite intersection of closed subsets is non-empty, then also the intersection of *any* collection of closed subsets of a compact set is non-empty.

Hint 2: Your proof should *NOT* assume that such preferences can be represented by a real-valued function. Only rely on this auxiliary result if you cannot prove it otherwise.

Question 2:

(a) Consider the economy:

$$(u^i, e^i, \theta^{ij}, Y^j)_{i \in I, j \in J}.$$

Define competitive equilibrium for an economy with production.

(b) Define what is a quasi-equilibrium with transfers.

(c) State the second welfare theorem for a general economy with production stated in class. Explain as well as you can why we need to relax the definition of competitive equilibrium to 'quasi-equilibrium.' A graphical illustration with a well-labeled graph is welcome.

(d) Provide conditions that guarantee that at least an equilibrium exists. It suffices to state the theorem we proved in class for an economy with production.

(e) Consider an economy in which consumer i has the strictly increasing and strictly quasiconcave utility function $u^i : \mathbb{R}^l_+ \to \mathbb{R}$, endowment vector $\omega^i \in \mathbb{R}^l_+$, $i = \{1, 2, \ldots, n\}$ and in which $Y_j \subset \mathbb{R}^l$ denotes firm j's production set, $j = \{1, 2, \ldots, m\}$. Suppose that $(\hat{x}, \hat{y}) = (\hat{x}^1, \ldots, \hat{x}^n), (\hat{y}^1, \ldots, \hat{y}^m)$ is a Pareto efficient allocation for this economy, where $\hat{x}^i \gg 0$ denotes the bundle going to consumer i and $\hat{y}^j$ denotes j's production vector. Construct a new economy by replacing the endowment vector $\omega = (\omega^1, \ldots, \omega^n)$ with the new endowment vector $(\hat{x}^1, \ldots, \hat{x}^n)$ and by replacing the production set with the new production set $\hat{Y}_j = Y_j - \{\hat{y}^j\}$. Show that if this economy possesses a competitive equilibrium, then in the new economy:

i. the bundle demanded in equilibrium by each consumer i is $\hat{x}^i$ and

ii. each new firm earns zero profits.

Question 3:

(a) Construct a single 2×2 normal-form game that simultaneously satisfies all four of the following properties:

i. The game is not solvable by weak dominance (at least one player does not have a weakly dominant strategy).

ii. The game is solvable by iterated weak dominance (so that only one pure strategy per player remains).

iii. In addition to the iterated weak dominance solution (which is a Nash equilibrium), there is a second pure-strategy Nash equilibrium.

iv. Both players strictly prefer the second equilibrium to the first.

(b) Consider the following game:

	L	R
U	$2,2$	$1,5$
D	$5,1$	$0,0$

Derive a correlated equilibrium that places positive probability on every pure strategy profile except (D, R), and that maximises the probability that the profile (U, L) is played.

Question 4: Consider a second price auction for a single object with two bidders and an up-front participation fee f. Bidder i's valuation, v_i, is iid uniformly on $[0, 1]$. The seller's valuation is zero. The timing is as follows. First, bidders privately observe their valuations and the seller offers a second price auction with a participation fee $f > 0$ which is a fee that the bidders have to pay if they participate in the auction, independently of the outcome of the auction. Second, given the auction format, the bidders have to simultaneously decide whether or not to participate, and if so, how much to bid. Placing a big equal to zero is equivalent to not participating in the auction. Thus, a bid equal to zero incurs no entry fee (but also excludes the bidder from winning the object), while every positive bid incurs an entry fee. Ties are broken by fair lottery.

(a) Describe the payoff function of bidder i as a function of an arbitrary pair of bids (b_i, b_j). (Careful: what happens if one of the players bids zero?)

(b) Define the notion of a pure strategy and the notion of a pure strategy Bayes-Nash equilibrium for the auction game with the entry fee. Describe the expected payoff of the auctioneer as a function of the bidding strategies and the entry fee.

(c) Compute a symmetric pure strategy Bayes-Nash equilibrium where bidder i bids her own valuation if her valuation exceeds a threshold v^*, and where otherwise she does not participate in the auction.

(d) Compute the seller's expected revenue from the auction with an arbitrary entry fee $f > 0$.

(e) Which entry fee maximises the expected revenue of the seller? Is the resulting allocation ex-post efficient?